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Master Thesis
in Quantitative Finance

Performance Analysis of Structured Financial Instruments for Different Risk Profiles under Various Market Scenarios

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List of Abbreviations

T	Expiration time
t	Current time
K	Strike price, exercise price
r_f	Risk-free rate
μ	Mean
σ	Volatility, standard deviation
S_0	Price of underlying asset in $t = 0$
S_T	Price of underlying asset at maturity
S_t	Current price of underlying asset
W	Wiener Process
B	Barrier
a	Subscription ratio
C_0	Value of European call option in $t = 0$
P_0	Value of European put option in $t = 0$
BL	Bonus level of Capped Bonus Certificate
CAP	Upper limit of Capped Bonus Certificate
CBC	Capped Bonus Certificate
ES50	Euro Stoxx 50
GBM	Geometric Brownian Motion
NV	Nominal value
PoS	Probability of Success
PRIIP	Packaged Retail and Insurance-based Investment Product
RCB	Reverse Convertible Bond
SDE	Stochastic Differential Equation
SR	Sharpe Ratio
VaR	Value at Risk
VEV	VaR-Equivalent Volatility

1. Introduction

The persistent low-interest environment triggers investors to look for more profitable investments than plain bank deposits. Considering the large number of existing investment products in the capital market, it is not easy for retail investors to find a tailored financial product which reflects their individual risk profile and their expected returns in a proper way.

As an opportunity, investors can consider investing in structured financial products such as certificates. According to the German Derivatives Association (DDV 2013), certificates provide profit opportunities in all market situations and suitable products for every risk propensity. The differentiated risk and payout profiles can be realized by combining fundamental securities, such as derivatives, equities, indices, bonds, currencies or commodities. Attractive risk-opportunity profiles and access to complex asset classes are thus made possible.

This study shows that an investment in a reverse convertible bond or in a capped bonus certificate can facilitate the alignment of the investor's risk appetite and that a positive return can be realized with a high probability.

Due to the complexity and non-transparency of certificates, these products are not easily understandable even for experienced retail investors. This study aims to enhance transparency regarding the return potentials and the related loss risks associated with structured financial products, in particular of reverse convertible bonds and capped bonus certificates, in various market scenarios of the underlying asset.

For this purpose, the mathematical framework of Itô calculus and Monte Carlo simulations are used as fundamental tools. The simulated performances consider different risk profiles in accordance with the EU-PRIIPs-Regulation (2017) of potential investors as well. The Euro Stoxx 50 index is taken as the relevant underlying asset for the structured products under consideration. The historical data required for the Monte Carlo simulations and the determination of the Value at Risk equivalent volatility are taken from Ariva.de.

The following chapter 2 explains the basic fundamentals of mathematical finance and risk management used for the performance analysis of the certificates. This includes options theory, Itô calculus, Monte Carlo method and Value at Risk. Chapter 3 deals with the structure and the basic functioning of reverse convertible bonds and capped bonus certificates. Furthermore, a method to evaluate the payout profiles of these complex products is provided. In chapter 4, the design of the empirical study is outlined, including the explanation of the different risk profiles and market scenarios considered. In addition, the results of the performance analysis are illustrated and discussed.

2. Methodology

In order to analyze the performance of structured financial products, in particular reverse convertible bonds and bonus cap certificates, sound knowledge of mathematical finance and stochastic processes is required.

In this chapter, we will discuss some essential background and methodology for the understanding of structured financial products and this study in particular. The payout profile of certificates can be replicated by a combination of the underlying asset and one or more optional components. Therefore, the fundamental technical functionality of standard options will be characterized beforehand.

2.1. Options

An option is a contract which grants the holder the right to buy or sell an underlying asset at a predetermined price (Seydel 2017: 1). Options, which can be exercised only at the expiry date T, are called European options (Asiri 2018: 5). If an exercise of the option is possible anytime during its lifetime, the option is called American option. In this study, only European options are relevant for the discussion since all investigated certificates can solely be exercised at maturity.

The risk profile according to DDV (2017: 14) is asymmetric since the buyer (long position) of an option has the right to choose (not the obligation) whether she exercises the option or not, whereas the seller (short position) has to accept a potential exercise in any case. Therefore, they belong to conditional forward transactions. In case of the exercise of a call option, the writer has to deliver the underlying asset for the strike price K (DDV 2017: 14). If a put option is exercised, the writer has to buy the underlying at the amount of the strike price. According to Seydel (2017: 1), the writer will compensate the loss risks through appropriate hedging strategies. The underlying asset of an option can be a stock, an index, a commodity or a currency.

Basically, we can distinguish four different risk-reward profiles for options on which every more complex option combination is built on: buying/selling a call and buying/selling a put (Schmidt 2014: 194). The opportunities and risks depend on the strike price and the break-even point.

2.1.1. Call Options

The first strategy is to buy a call (long call). In this case, the buyer is convinced that the price of the underlying will rise (Bloss 2017: 261). She acquires the right to buy the underlying asset via a call option. The loss potential is limited to the option premium, though she receives the chance to participate in unlimited share price increases, which is

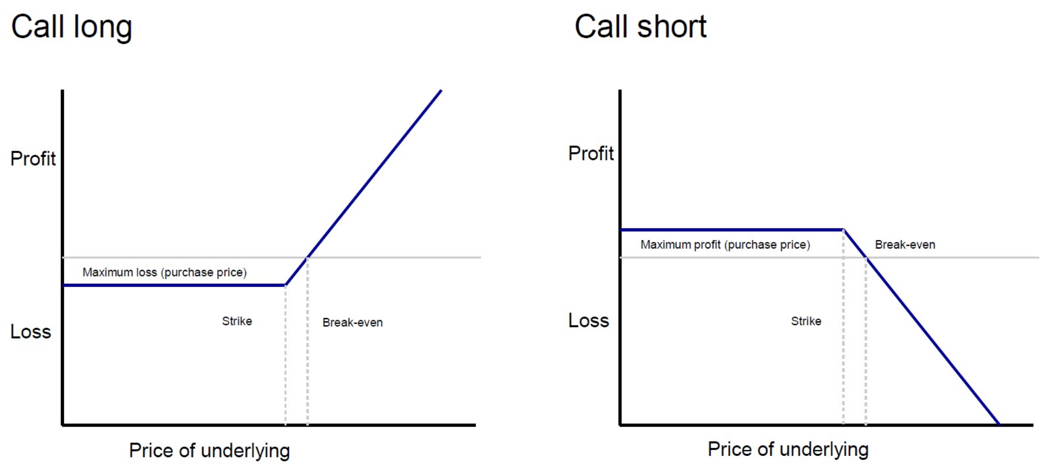

Figure 1: Risk-reward Profile Long Call and Short Call. Source: Own diagram based on DDV (2017: 16).

illustrated on the left-hand side of Figure 1. If the market price of the underlying settles below the strike at maturity, the buyer will not exercise the call option (Kallsen 2015: 54). If the share price is above the strike, the payoff is the difference of both (Sengputa 2004: 309). Following Kallsen (2015: 54), the value of a call option can be depicted as a random variable X, which represents the payoff at time T depending on the share price at maturity and the strike price.

$$X = (S_T - K)^+ = max(S_T - K, 0) \tag{1}$$

A second strategy for an investor is to sell a call (short call). In this case, she reckons on a constant or slightly decreasing price scenario and wants to generate additional income through the option premium (Bloss 2017: 261). The maximum profit is limited in the amount of the received option premium. The risk is characterized by rising market prices since the underlying has to be delivered at the strike price. A large price increase leads into the loss area (Schmidt 2014: 196). In accordance with Bloss (2017: 262), this risk can be minimized by holding the underlying asset in the portfolio prior to the conclusion of the forward transaction.

2.1.2. Put Options

With regard to put options an investor also has two possible strategies. By buying a put (long put), the investor expects a significant fall in the market price. With this strategy, she speculates actively against falling prices and attempts to profit from the downward market trend, or she just intends to hedge against a decline of the share price because she

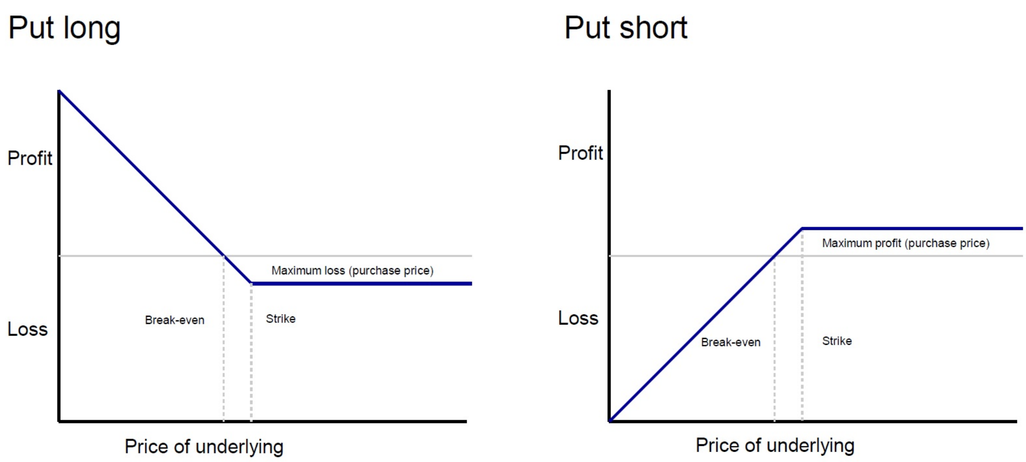

Figure 2: Risk-reward Profile Long Put and Short Put. Source: Own diagram based on DDV (2017: 17).

has the stock in her portfolio (Bloss 2017: 262). On the one hand, the maximum loss is limited to the paid option premium, which is illustrated in Figure 2 on the left-hand side. On the other hand, the profit is bounded to a maximum, since every asset can only fall to a value of zero.

The value of a put option at maturity T can be likewise expressed as a random variable X, depending on the share price and the strike price.

$$X = (K - S_T)^+ = max(K - S_T, 0) \tag{2}$$

In case of selling a put (short put), the investor anticipates constant or slightly increasing prices in the specific asset market (Schmidt 2014: 196). The option premium depicts the maximum profit. The loss is limited to the strike at maximum, since under circumstances of falling prices she has to buy the underlying asset at the fixed strike price. Therefore, the investor is exposed to the risk of a partial or a complete loss of the underlying.

2.1.3. Barrier Options

Options, for which the contract expires if the underlying share price reached a certain level (barrier), are called barrier options (Irle 2012: 174). The payout of barrier options depends not only on $S(T)$ but on the entire path $S(t)_{t \in [0,T]}$ of the underlying asset. Therefore, they are path dependent and belong to exotic type of options. Depending on the position of the barrier in relation to the underlying price, a distinction is made between up-options and down-options (Löhr and Cremers 2007: 21).

As an example, consider a European down-and-out-put with duration T, exercise price

Transaction	Current Value	Value at expiration $S_T \leq K$	$S_T > K$
European call			
Buy call	C_0	0	$S_T - K$
Synthetic European call			
Buy put	P_0	$K - S_T$	0
Buy underlying asset	S_0	S_T	S_T
Issue bond	$-Ke^{-rT}$	$-K$	$-K$
Total	$P_0 + S_0 - Ke^{-rT}$	0	$S_T - K$

Table 1: European Call and Synthetic European Call. Source: Müller (2016: 6).

K and barrier B.

$$X = (K - S_T)^+ \mathbb{1}_{\forall S_t > B} \tag{3}$$

The equation (3) demonstrates the payout at maturity of this specific barrier option. Following Irle (2012: 174), this put expires if the price of the underlying reaches or falls below the level B. If the threshold is not touched or fallen short of, the payoff equals that of a classical put option.

2.1.4. Put-Call Parity

In modeling the payoff profile and the price of structured financial instruments, the put-call parity is useful. According to Müller (2016: 6), the payoff structure of a European long call with exercise price K can be replicated by a portfolio containing a European long put with the same strike, a long position of the underlying asset with current price S_0 and a short position of a bond with nominal value K. The payoff at maturity T (value at expiration) will be the same for both investments, which is outlined in Table 1.

$$C_0 = P_0 + S_0 - Ke^{-rT} \tag{4}$$

Since the European long call and the portfolio (synthetic European call) provide the same payoff at time T, they should have the same fair price at time $t = 0$ in an arbitrage-free market (Müller 2016: 6). This relation is called put-call parity and is demonstrated in equation (4). According to Asiri (2018: 7), an arbitrage opportunity is a riskless opportunity to earn money resulting from mispriced securities.

2.2. Preliminaries

In this chapter some basic essentials of mathematics are described for a better understanding of Itô calculus and of some general assumptions in financial mathematics.

In this context, theorems of probability theory regarding the normal distribution and the log-normal distribution play a decisive role.

2.2.1. Normal Distribution of Returns

A standard assumption in financial engineering is that returns over a small period of time are normally distributed and returns over two non-overlapping periods are independent (Hull 2018: 313).

A continuous random variable X is normally or Gaussian distributed with mean μ and variance σ^2 (notation: $X \sim N(\mu, \sigma^2)$) if the probability density function is defined by (Asiri 2018: 2):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}. \tag{5}$$

The random variable X can be expressed as

$$X = \mu + \sigma Z, \tag{6}$$

where Z is a standard normal variable with $Z \sim N(0,1)$.

The assumption of normally distributed returns underlies the notable Black-Scholes model for option valuation since the possible future stock prices at the end of a period are expected to be log-normal distributed (Black and Scholes 1973: 640). It is not a perfect representation of the reality, but nevertheless, it is an acceptable approximation in a practical setting according to Schmidt (2014: 48). In real life, very small and very large changes in the asset price occur more frequently than what would be anticipated by the normal distribution.

2.2.2. Log-Normal Distribution of Stock Prices

From log-normally distributed share prices follows that the percentage changes are normally distributed, since the logarithm of a log-normally distributed random variable is normally distributed (Schmidt 2014: 49).

This is also in accordance with the assumptions of the Black-Scholes model (Black and Scholes 1973: 640). Random variables drawn from this distribution can not take negative values, which is in line with the reality and an important feature in modeling stock prices.

A continuous, positive random variable Y is log-normally distributed with parameters

μ and σ (notation $Y \sim LN(\mu, \sigma^2)$) if the transformed variable

$$ln(Y) = \mu + \sigma Z \tag{7}$$

is normally distributed with standard normal variable Z (Asiri 2018: 3).

2.3. Itô Calculus

For the performance analysis of structured products, assumptions about the fundamental price behavior of assets are crucial to simulate future outcomes. The price development of the underlying asset can be seen as an uncertain course of a random variable through time (Hull 2018: 313).

In accordance with Iacus (2011: 104), Bloss (2017: 31) and Hull (2018: 313), financial time series can be imagined as stochastic processes, for which only the present value of the variable is relevant for predicting the future. Historical data and further background information are not needed to forecast the future evolution of the variable. Therefore, it is assumed that share prices can be described adequately and comprehensively via a Markov process (Bloss 2017: 31).

2.3.1. Wiener Process

A continuous-time stochastic process belongs to the group of random variables $X(t)$, which are defined for continuous time t, for example in $0 \leq t \leq T$ (Seydel 2017: 27). $X(t)$ or X_t is the realization of the stochastic process.

Following Seydel (2017: 27), there are two particular characteristics which can constitute a stochastic process. First, it can be a Gauß process, where $X(t)$ is normally distributed for every t. Second, it can be a Markov process, where only the current value of X is relevant for the future behavior since all information of the past is already included in the current value.

A standard Brownian motion or Wiener process is both, a Gauß and a Markov process. The variable follows a process that can be described by a standard normal distribution. It can be defined as a continuous stochastic process $\{W(t), 0 \leq t \leq T\}$ with the following properties according to Asiri (2018: 25), Glasserman (2004: 79), and Iacus (2011: 104):

(i) $W(0) = 0$

(ii) the mapping $t \mapsto W(t)$ is a continuous function on $[0, T]$

(iii) the increments $\{W(t_1) - W(t_0), W(t_2) - W(t_1), \ldots, W(t_n) - W(t_{n-1})\}$ are independent for any n and any $0 \leq t_0 < t_1 < \cdots < t_n \leq T$

(iv) $W(t) - W(s) \sim N(0, t-s)$ for any $0 \leq s < t \leq T$

(v) $W(t) \sim N(0,t)$ for $0 < t \leq T$

Therefore, the Wiener process $W(t)$ describes the evolution of a normally distributed random variable through time with drift rate 0 and diffusion coefficient 1.

2.3.2. General Brownian Motion

For general constant parameters μ and $\sigma > 0$, the process $X(t)$ is called Brownian motion with drift μ and diffusion coefficient σ^2 (Glasserman 2004: 80). Since $\frac{X(t)-\mu t}{\sigma}$ is a standard Brownian motion, we can write X as:

$$X(t) = \mu t + \sigma W(t). \tag{8}$$

It follows from the characteristics of a Wiener process that $X(t) \sim N(\mu t, \sigma^2 t)$. Furthermore, X solves the stochastic differential equation (SDE):

$$\mathrm{d}X(t) = \mu \mathrm{d}t + \sigma \mathrm{d}W(t). \tag{9}$$

Consequently, variable X follows a path with deterministic expected change μ per unit of time and with additional stochastic movement (diffusion) $\sigma \mathrm{d}W(t)$ on the covered distance (Bloss 2017: 33).

The differential equation (9) does not make sense in ordinary calculus, since it is mathematically not well defined, but following Kallsen (2017a: 20) it can be interpreted as an integral equation of the form

$$X(t) = X(0) + \int_0^t \mu \mathrm{d}s + \int_0^t \sigma \mathrm{d}W(s), \tag{10}$$

where $X(0)$ is an arbitrary constant for the value of X at time $t = 0$. Processes of this form are called Itô processes. As well as the standard Brownian motion, the process X has continuous sample paths and independent increments (Glasserman 2004: 80). Each increment $X(t) - X(s)$ is normally distributed with mean

$$E(X(t) - X(s)) = \int_s^t \mu \mathrm{d}u \tag{11}$$

and variance

$$Var(X(t) - X(s)) = \int_s^t \sigma^2 \mathrm{d}u. \tag{12}$$

2.3.3. Itô Process

The framework of both, the Wiener process and the general Brownian motion, can be described by means of an Itô process. Modeling financial time series with ordinary calculus is not appropriate, because stochastic fluctuations are neglected (Seydel 2017: 32). Instead, Itô processes form the base for stochastic calculus and the modeling of share prices.

An Itô process can be written as the SDE

$$X(t) = X(0) + \int_0^t a(X(s), s)\mathrm{d}s + \int_0^t b(X(s), s)\mathrm{d}W(s) \tag{13}$$

or symbolically as

$$\mathrm{d}X(t) = a(X(t), t)\mathrm{d}t + b(X(t), t)\mathrm{d}W(t). \tag{14}$$

The change of a variable $X(t)$ is composed of a Wiener process $\mathrm{d}W(t)$ and the functions a and b, which depend on $X(t)$ and t (Hull 2018: 311). Therefore, the drift and the variance rate of X can be a function of both, X itself and the time.

A Wiener process itself is a particular Itô process with $a = 0$ and $b = 1$ (Seydel 2017: 33). Analogously to a Brownian motion, the variable $X(t)$ has a drift a and a variance of b^2.

2.3.4. Itô Lemma

In stochastic calculus, the rules for differentiation and integration differ from those in ordinary calculus (Asiri 2018: 32). To make calculations with an Itô-integral, it requires suitable calculation rules. A fundamental foundation of stochastic processes is Itô's lemma, with which one can derive specific solutions for SDEs. Asiri (2018: 32) emphasizes, that for stochastic variables it is as important as a Taylor series is for deterministic variables.

Let $X(t)$ be a stochastic process as in equation (14), and suppose $f(x,t)$ is a function with continuous $\frac{\partial f}{\partial x}$, $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial f}{\partial t}$. Then, $f(X(t),t)$ also follows an Itô process with the same Wiener process $W(t)$. According to Seydel (2017: 41) and Kallsen (2017a: 21), Itô's lemma reads as:

$$\mathrm{d}f(X(t), t) = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} a + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} b^2 \right) \mathrm{d}t + \frac{\partial f}{\partial x} b \mathrm{d}W(t) \tag{15}$$

or

$$\mathrm{d}f(X(t)) = \left(f'(X(t))a + \frac{1}{2} f''(X(t))b^2 \right) \mathrm{d}t + f'(X(t))b\mathrm{d}W(t) \tag{16}$$

if the function f depends only on $X(t)$. The entire proof of Itô's formula is outlined in Arnold (1973: 108).

2.3.5. Geometric Brownian Motion

The general Brownian motion implies a constant expected drift, as well as a constant variance rate (Bloss 2017: 35). Thus, if the underlying variable of the process should describe the evolution of a share price in time, a constant absolute change of the price would be presumed in the case of the general Brownian motion.

However, in reality, investors request a percentage return which is independent of the share price level. As a consequence, the assumption of constant price changes must be replaced through the assumption of constant expected percentage returns, which are defined as the quotient of the expected price change and the share price. It follows that at any given time point t, the drift of the share price S is expected to be $\mu S(t)$. The constant parameter μ corresponds to the expected (annualized) return of the stock.

The percentage change (return) $\frac{\mathrm{d}S}{S}$ of a stock in the time interval dt consists of a deterministic drift part $\mu \mathrm{d}t$ and the stochastic fluctuations $\sigma \mathrm{d}W$ (Seydel 2017: 34). The geometric Brownian motion (GBM) is defined by the stochastic differential equation:

$$\frac{\mathrm{d}S(t)}{S(t)} = \mu \mathrm{d}t + \sigma \mathrm{d}W(t) \tag{17}$$

or

$$\mathrm{d}S(t) = \mu S(t)\mathrm{d}t + \sigma S(t)\mathrm{d}W(t) \tag{18}$$

with W a standard Brownian motion. This SDE is linear in $x = S$, with $a(S,t) = \mu S$ and $b(S,t) = \sigma S$. As stochastic integral equation, the GBM can be expressed as:

$$S(t) - S(0) = \int_0^t \mu S(s)\mathrm{d}s + \int_0^t \sigma S(s)\mathrm{d}W(s). \tag{19}$$

The parameter μ is the annualized mean and σ represents the volatility of the stock price (Glasserman 2004: 4). According to Schmidt (2014: 47), the volatility is a measure for the average yearly percentage stock price changes around their mean. Consequently, the volatility is the annualized standard deviation of the returns. The conversion from daily to yearly standard deviation is feasible using the formula

$$\sigma_{yearly} = \sigma_{daily}\sqrt{\text{number of periods}} = \sigma_{daily}\sqrt{256}. \tag{20}$$

To express the evolution of stock prices via geometric Brownian motion constitutes the reference model on which Black and Scholes (1973) and Merton (1973) established their assumptions for the valuation of derivatives and their theory in option pricing. In financial mathematics, the GBM is the most fundamental model for the value of financial assets.

The solution of the SDE in equation (18) can be derived by means of Itô's lemma and results in

$$S(t) = S(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right) \tag{21}$$

or generally, if $u < t$ then

$$S(t) = S(u)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t-u) + \sigma\Big(W(t) - W(u)\Big)\right). \tag{22}$$

$S(0)$ is the initial price of the stock and is assumed to be known.

Following Grüne (2011: 32), consider the SDE $\mathrm{d}Z(t) = a\mathrm{d}t + b\mathrm{d}W(t)$, with $a \equiv 0$ and $b \equiv 1$, the Itô formula can be applied on the function $Y(t) = f(Z(t),t)$ with

$$f(x,t) = S(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma x\right). \tag{23}$$

The partial derivatives yield

$$\frac{\partial f(x,t)}{\partial t} = f(x,t)(\mu - \frac{1}{2}\sigma^2), \quad \frac{\partial f(x,t)}{\partial x} = f(x,t)\sigma, \quad \frac{\partial^2 f(x,t)}{\partial x^2} = f(x,t)\sigma^2. \tag{24}$$

Since $Y(t) = f(Z(t),t) = S(t)$ (because $Z(t) = W(t)$), it follows from Itô lemma that

$$\mathrm{d}S(t) = \mathrm{d}Y(t) = \left(S(t)\left(\mu - \frac{1}{2}\sigma^2\right) + \frac{1}{2}\sigma^2 S(t)\right)\mathrm{d}t + S(t)\sigma\mathrm{d}W(t) \tag{25}$$

$$= \mu S(t)\mathrm{d}t + \sigma S(t)\mathrm{d}W(t). \tag{26}$$

Consequently, equation (21) is indeed the solution for the stochastic differential equation in (18).

The logarithm of the stock price depends linearly on a normally distributed Wiener process and for this reason, it is itself normally distributed (Glasserman 2004: 4). The logical conclusion from this is that the stock price has a log-normal distribution. Therefore, if $S \sim GBM(\mu,\sigma^2)$ we have

$$\frac{S(t)}{S(0)} \sim LN\left(\left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right) \tag{27}$$

with

$$E(S(t)) = S(0)e^{\mu t} \tag{28}$$

$$\text{and } Var(S(t)) = S(0)^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right) \tag{29}$$

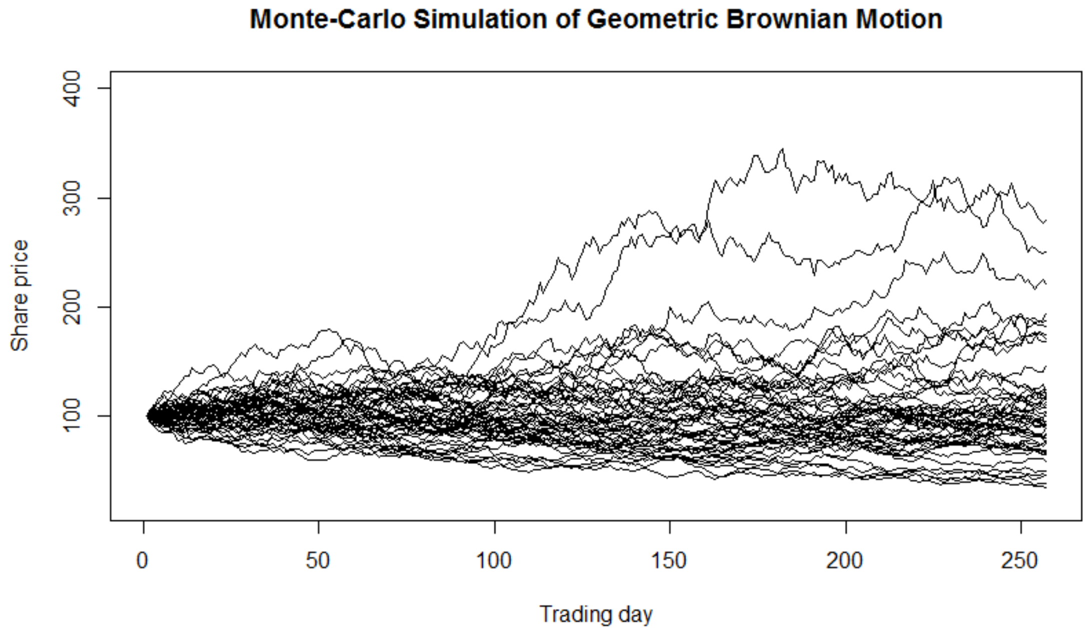

Figure 3: Monte-Carlo Simulation of Geometric Brownian Motion, with $S(0) = 100$, $\mu = 0.1$, $\sigma = 0.4$, $N = 256$ and 50 paths. Source: Own diagram.

and

$$ln(S(t)) - ln(S(0)) \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right) \tag{30}$$

or

$$ln(S(t)) \sim N\left(ln(S(0)) + \left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right). \tag{31}$$

It can thus be concluded that a stochastic process $S(t)$ is a GBM if the logarithm of this process is a general Brownian motion with initial value $ln(S(0))$, $a \equiv (\mu - \frac{1}{2}\sigma^2)$ and $b \equiv \sigma$. Hence, a GBM is just an exponentiated Brownian motion (Glasserman 2004: 94).

Whereas an ordinary Brownian motion can also take negative values, a log-normally distributed variable can only take values between 0 and ∞, which is why the process is ideal for the distribution assumption for future share prices. Negative values for stock prices or other limited liability assets would be an undesirable feature in a predictive model (Glasserman 2004: 93).

Furthermore, for the GBM the percentage changes $\frac{S(t_2)-S(t_1)}{S(t_1)}, \frac{S(t_3)-S(t_2)}{S(t_2)}, \ldots, \frac{S(t_n)-S(t_{n-1})}{S(t_{n-1})}$ are independent for $t_1 < t_2 < \cdots < t_n$, rather than the absolute changes. In modeling assets prices these properties are essential to get reasonable values.

According to Glasserman (2004: 94), one can utilize the feature that the increments of a Wiener process W are independent and normally distributed, such that a recursive procedure is appropriate when it comes to the simulation of stock prices S at time $0 =$

$t_0 < t_1 < \cdots < t_n$. The formula for the path simulation of the asset price is given by

$$S(t_{i+1}) = S(t_i)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} Z_{i+1}\right), \qquad (32)$$
$$i = 0, 1, \ldots, n-1$$

with $Z_1, Z_2, \ldots, Z_n$ independent standard normal random variables, i.e. $Z \sim N(0,1)$.

Figure 3 illustrates an example simulation according to equation (32) of 50 feasible future share price paths within one trading year (256 trading days) and with starting value $S(0) = 100$. The underlying asset is expected to have an average return of $\mu = 0.1$ and a volatility of $\sigma = 0.4$. Because of the positive drift term, the asset price at time $T = 256$ is most likely above the initial value of $S(0) = 100$. Only in a few cases, the price falls gravely below the starting value.

2.4. Monte Carlo Simulation

The Monte Carlo simulation is a frequently used method in finance and economics to find an approximate numerical solution with the help of probability theory for analytically unsolvable tasks (Bloss 2017: 252). For example, it can be used to simulate paths of a stochastic process to illustrate the development of potential future outcomes.

By generating a very large quantity of random variables, using a pseudorandom number generator on the computer, the objective function can be calculated and the estimator can be determined.

In financial engineering, the future probability distribution of the underlying variable is of importance. It can be calculated by employing a Monte Carlo simulation, dividing a time interval (e.g. one year) into many small time steps (e.g. 256 trading days), and then randomly generating potential paths for the variable (Hull 2018: 313). The distribution of the underlying variable can then be estimated from the distribution of the simulated outcomes. According to Bloss (2017: 252), the Monte Carlo method can approximately determine the optimum of this specific objective function.

Following the steps of Kallsen (2017a: 47) and Iacus (2011: 159), suppose we are interested in the evaluation of expectations in the form $V = E(f(X))$ numerically, where X denotes a random variable and f some known function. It is assumed that we can simulate X on the computer, i.e. it is possible to draw N realizations of independent and identically distributed random variables $X_1, \ldots, X_N$ from the given distribution of X. As an approximation for the true value of V we then can use the empirical mean

$$\hat{V}_N = \frac{1}{N}\sum_{n=1}^{N} f(X_n). \qquad (33)$$

$\hat{V}_N$ is an unbiased estimator for the expectation $V = E(f(X))$ and according to the law of large numbers it converges to $V = E(f(X))$ for $N \to \infty$. Due to the central limit theorem, the Monte Carlo estimator $\hat{V}_N$ is asymptotic normally distributed such that

$$\hat{V}_N \xrightarrow{d} N\left(E(f(X)), \frac{1}{N} Var(f(X))\right), \tag{34}$$

which implies that the standard deviation of the estimator is

$$\sigma(\hat{V}_N) = \sqrt{Var(\hat{V}_N)} = \frac{1}{\sqrt{N}} \sigma(f(X)). \tag{35}$$

Therefore, the simulation has a rate of convergence to the true value of $\frac{1}{\sqrt{N}}$. By increasing the number of simulations by the factor of four, the error can be halved on average.

The property of asymptotic normality can be used to determine a confidence interval for the Monte Carlo estimator. With probability of 95%, the true value V lies within the interval

$$\left[\hat{V}_N - 1.96\sqrt{\frac{\hat{\sigma}_N^2(f(X))}{N}}, \hat{V}_N + 1.96\sqrt{\frac{\hat{\sigma}_N^2(f(X))}{N}}\right], \tag{36}$$

where $\hat{\sigma}_N^2(f(X))$ is the approximation of the variance of $f(X)$. Since even the expectation of $f(X)$ is unknown, the variance is more than likely also unknown. It can be approximated by the sample variance, which is an unbiased and consistent estimator of the variance:

$$\hat{\sigma}_N^2(f(X)) = \frac{1}{N-1} \sum_{n=1}^{N} \left(f(X_n) - \hat{V}_N\right)^2. \tag{37}$$

To conclude, with the Monte Carlo method we can get an arbitrarily precise estimator of our objective function by simulating sufficiently often.

2.5. Value at Risk

The Value at Risk (VaR) is a risk measure which is frequently used in practice to assess the risk position of a portfolio. In general, it is defined as the absolute loss (or negative profit) of a portfolio, which is not exceeded in a predefined period of time with a certain probability (Bloss 2017: 63). In accordance with Christoffersen (2003: 48), the absolute €VaR for a probability $p \in (0,1)$ is defined as

$$Pr(€Loss > €VaR) = p, \tag{38}$$

i.e. the €*Loss* of the investment will exceed the €*VaR* with probability p. The percentage portfolio return at time t can be defined as R_t such that we can write the €*Loss* as

$$€Loss = -(V_{t+1} - V_t) = -V_t \times R_{t+1} \tag{39}$$

where V_t is the current market value of the portfolio. By substituting this relationship into the definition of the absolute €VaR and given the cumulative distribution function of the profit and loss distribution is continuous, the percentage VaR for level $p \in (0,1)$ is defined as

$$Pr(R_{t+1} < VaR) = p, \tag{40}$$

with

$$VaR \equiv -\frac{€VaR}{V_t}. \tag{41}$$

Since the true profit and loss distribution of the investment is unknown, an appropriate estimate for the VaR value is required. Following Kallsen (2017b: 31), the estimate $\widehat{VaR}$ can be determined via

$$\widehat{VaR} := \hat{q}_p(F_N) := r_{[p\times N]+1:N}, \tag{42}$$

where we use the estimated probability distribution of the returns generated by the Monte Carlo method from the previous chapter. In this context, the function $F_N : \mathbb{R} \mapsto [0,1]$ is the empirical distribution function of the simulated returns $r_1, r_2, \ldots, r_N$ defined by

$$F_N(x) = \frac{\text{number of sample elements} \leq x}{N} = \frac{1}{N}\sum_{k=1}^{N} \mathbb{1}_{[r_k,\infty)}(x) \tag{43}$$

and $\hat{q}_p(F_N)$ is the empirical p-quantile of F_N for $p \in (0,1)$ (Kallsen 2017b: 25). The ordered random variables $r_1, r_2, \ldots, r_N$ are denoted by $r_{1:N} \leq r_{2:N} \leq \cdots \leq r_{N:N}$ and $[x] := max\{n \in \mathbb{N} : n \leq x\}$.

Therefore, the Monte Carlo estimate $\widehat{VaR}$ is the $[p \times N] + 1$-largest observation of the ordered return sample or, in other words, the empirical p-quantile.

2.6. Probability of Success

Despite its simplicity, the probability of success is a crucial figure for the comparison and assessment of different risk positions. It is defined as the probability that a specific investment strategy results in a positive return in the future, i.e.

$$PoS(R_{t+1}) = Pr(R_{t+1} > 0). \tag{44}$$

The true distribution function of future returns is unknown, such that we have to rely on the simulated returns $r_1, r_2, \ldots, r_N$ of the Monte Carlo simulation. The estimated probability of success $\widehat{PoS}(R_{t+1})$ can be calculated by means of the sample returns and it is defined by

$$\widehat{PoS}(R_{t+1}) := \frac{1}{N} \sum_{n=1}^{N} \mathbb{1}_{r_n > 0}. \tag{45}$$

2.7. Sharpe Ratio

The Sharpe Ratio is an economic figure which demonstrates the relationship between the excess return and the included risk of an investment. It is defined as the expected return in excess of the risk-free interest rate per unit of risk, where the standard deviation of the return is taken as the risk measure. In general, it is a method for determining the risk-adjusted return, that can be used to compare different investment strategies.

Following Sharpe (1966: 122), it can be calculated with the formula

$$SR = \frac{E(R_i) - r_f}{\sigma(R_i)}, \tag{46}$$

where $E(R_i)$ is the expected return of portfolio i and $\sigma(R_i)$ depicts its volatility. It is assumed that an investor can lend and borrow any desired amount of money at the given risk-free rate r_f. Therefore, an investor will always prefer the investment with the highest Sharpe Ratio.

For the estimated Sharpe Ratio $\widehat{SR}$ we will again utilize the simulated return values $r_1, r_2, \ldots, r_N$ from the Monte Carlo simulation and calculate the sample mean

$$\hat{\mu}_i = \frac{1}{N} \sum_{n=1}^{N} r_{n,i} \tag{47}$$

and the sample standard deviation

$$\hat{\sigma}_i = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} (r_{n,i} - \hat{\mu}_i)^2}, \tag{48}$$

which can be substituted in

$$\widehat{SR} = \frac{\hat{\mu}_i - r_f}{\hat{\sigma}_i}. \tag{49}$$

3. Structured Financial Instruments

In the last decades, there was a shift of investors demand from typical investment products towards financial instruments with more complex payoff structures than stocks, bonds or plain vanilla options (Szymanowska et al. 2009: 2). Generally, structured financial instruments are designed to facilitate access to such intricate positions in these elementary markets.

Structured products have the basic feature in common that they are all built through a combination of fundamental financial securities, forming complex cash flow patterns combined from fixed-income securities, equities and/or derivatives (Hernández et al. 2008: 3; Meyer 2013: 9). Even features of exotic options, such as long and short positions in barrier options, can serve as the basic framework for the payoff profile of these securities. The underlying asset can be a share, a basket of shares, an index, a commodity, a currency or a fond (Guidolin and Pedio 2015: 1; Löhr and Cremers 2007: 15).

According to Hernández et al. (2008: 4), an important benefit of combining several separate financial instruments into one single product is that transaction costs can be reduced. If the underlying asset is e.g. an index, a direct investment in the underlying is accompanied with a manual duplication of the index by buying all its components with prescribed weighting scheme (Meyer 2013: 10). In comparison to the direct investment in the underlying components, investing in the structured product would be consequently much more efficient.

Furthermore, the market can be completed by offering a variety of different payoff structures to meet the needs of the customers. It is feasible to depict highly customized risk-return profiles with differentiated payout at maturity defined through exact formulas published by the issuers (Guidolin and Pedio 2015: 1; Meyer 2013: 10). The differentiated risk and payout profiles of structured products are attainable by combining an underlying asset with one or more optional components (DDV 2017: 10). Those components react differently to changes in one or more parameters. Hence, the investor can gain in every market situation, depending on the design of the product.

Normally, retail investors have no or only limited access to exchanges of derivative financial instruments through their broker, especially in the case of over-the-counter (OTC) traded options, which are often an element of structured products (Löhr and Cremers 2007: 15; Meyer 2013: 10). Through the purchase of a structured product, the retail investor can receive access to such sophisticated trading strategies, besides the common investments in stocks, funds, or bonds. As the share prices of structured products are reduced to a relatively low level, smaller investments are possible. Therefore, the primary buyers of structured financial instruments, according to Meyer (2013: 10), are retail customers with insufficient funds, knowledge, and access opportunities to compose complex security combinations themselves.

There is a wide range of different product types and features in the market for structured instruments, which enables investors to find the appropriate product reflecting their desired risk-return profile (Meyer 2013: 11). Thus, there are products issued with various basic parameters and maturities. Issuers assume the function of a market-maker and provide permanently buying and selling prices for their products, in order to ensure sufficient liquidity in the secondary market (Löhr and Cremers 2007: 15; Burth et al. 2001: 31).

Equally noteworthy are potential downsides of structured instruments. Pursuant to Meyer (2013: 11), the investors carry no voting rights on matters of the corporate policy and usually, they have to waive any claims on occurring dividend payments if the underlying is a stock. The issuers often withhold dividends to finance special attributes of their products or they adjust product prices to the expectation of future dividend payments.

Certificates, like reverse convertible bonds (RCBs) and capped bonus certificates (CBCs), pertain to those derivatives and structured financial instruments. A certificate is a security that has the same legal form as a bond, so that potential payment difficulties of the issuer play an important role during the selection process of the certificate (Löhr and Cremers 2007: 15). In the case of a bankruptcy of the product supplier, the investor can suffer a total loss. Hence, the creditworthiness of the issuer is a crucial factor when it comes to picking the optimal certificate.

In the following subsections, the structure and the evaluation of the payment flow of reverse convertible bonds and capped bonus certificates will be discussed. The universal approach, which is often utilized to handle relatively complex financial constructions, especially for the valuation and the analysis of structured products, is called "evaluation by duplication" (Löhr and Cremers 2007: 16). The foundation of this approach is that structured financial securities exhibit an identical value and the same risk if they result in an identical cash flow pattern given constant surrounding conditions.

3.1. Reverse Convertible Bond

Reverse convertible bonds are bonds with a coupon payment which is located considerably above the current market interest rate (Wilkens and Scholz 2000: 171).

In exchange, the issuer has the right of decision at maturity to either disburse the nominal value of the security or to deliver a predefined quantity of shares of the underlying stock (Szymanowska et al. 2009: 1). In case of index-linked bonds, i.e. RCBs with an index as the underlying asset, generally a cash settlement in the amount of the value of the underlying index at maturity takes place (DDV 2017: 62).

Particularly, in times with low interest rates in the bond markets and high volatilities in the stock markets, RCBs constitute an attractive investment since they provide high

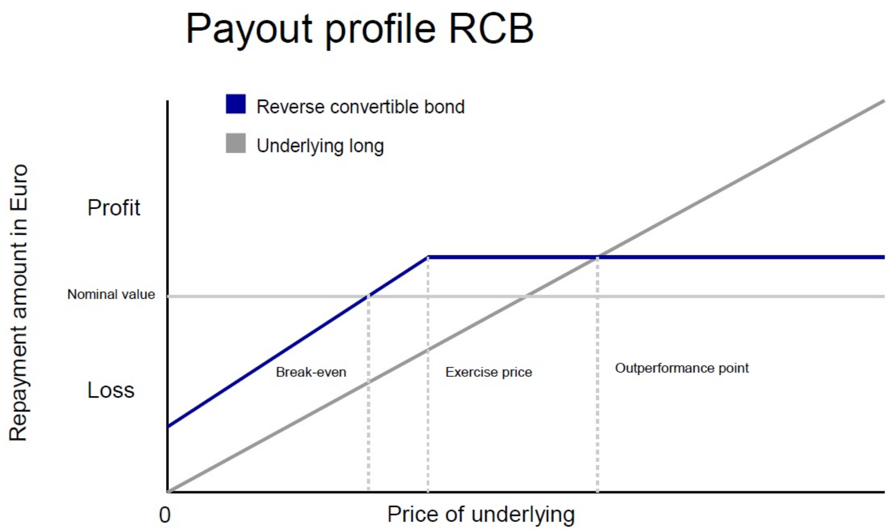

Figure 4: Payout Profile of Reverse Convertible Bond. Source: Own diagram based on DDV (2017: 65).

interest coupons. At expiry date, the issuer has to pay the interest coupon in any case, regardless of the performance of the underlying.

3.1.1. Structure

Reverse convertible bonds are regularly issued by large banks under the legal form of a bond with a duration mostly between 3-15 months (Löhr and Cremers 2007: 33). In contrast to stocks, they are issued with a nominal value and not as individual pieces (DDV 2017: 63). The smallest tradable denomination is generally located at a par value of 1000 Euro. Accordingly, the pricing takes place as a percentage quotation with regard to the par value, and not as the pricing of a share as Euro per item.

The investor of an RCB buys the high interest rate in exchange for the risk that she gets back a specified number of shares or the cash value of these shares, instead of the initial par value of the investment. The number of shares obtained by the investor defines the subscription ratio, which is equal to the notional amount divided by the strike price (Guidolin and Pedio 2015: 6).

If the price of the underlying asset is below the agreed critical value at maturity, the issuer will use the shares as amortization payment. This critical value has the same function as the strike price K from chapter 2. The disbursed cash amount or the value of the tendered shares is then calculated from the subscription ratio multiplied by the current price (Löhr and Cremers 2007: 34). If the price of the underlying share is listed at or over the particular exercise price at the expiry date, the issuer will pay back the bond at face value in cash.

The exercise price is obtained by calculating

$$K = \frac{\text{nominal value of RCB}}{\text{subscription ratio}} = \frac{NV}{a}. \tag{50}$$

Generally, it is set under the present share price at the issue date. The difference between the actual share price and the strike price serves thus as a risk buffer.

Figure 4 illustrates an exemplary payout profile of an RCB in comparison to a direct investment in the underlying stock. The maximum return of the RCB is ensured if the underlying is traded above the exercise price at the maturity date T. However, the maximum return is limited to the amount of the coupon payment, even if the underlying realizes immense price gains in a bull market. As long as the underlying share price does not rise above the outperformance point, the return of the RCB is higher compared to the direct investment.

The price at the maturity date at which the direct investment has a yield advantage in comparison to the RCB can be calculated via

$$\text{outperformance point} = \text{current share price} \times \left(1 + \frac{\text{Interest rate in \%}}{100}\right) \tag{51}$$

or equally via

$$\text{outperformance point} = \frac{\text{nominal value of RCB} + \text{coupon payments}}{\text{number of shares at direct investment}} \tag{52}$$

with

$$\text{number of shares at direct investment} = \frac{\text{nominal value of RCB}}{\text{current share price}}. \tag{53}$$

Thus, solely in a sustainable bull market the direct investment in the underlying is superior to the RCB (Löhr and Cremers 2007: 37).

The critical value of the underlying at which the RCB reaches the loss region can be calculated through the computation of the break-even point. Following Löhr and Cremers (2007: 34), it is defined as

$$\text{break-even point} = \frac{\text{nominal value of RCB} - \text{coupon payments}}{\text{subscription ratio}}. \tag{54}$$

This means that the investor will make a loss if the price of the underlying is located below the break-even point at maturity, since the risk buffer of the coupon payments is then exhausted. Hence, the risk of an investment in an RCB is characterized by sharply declining prices of the underlying asset.

The risk of loss is not bounded. In the case of a bankruptcy of the company, which has issued the underlying asset, the investor will suffer a total loss. Nevertheless, the negative return will not account for 100% of the investment, since the investor has received an above average interest payment during the period. Therefore, the losses of an RCB are cushioned through the obtained coupon payments, in comparison to the direct investment in the underlying, but the losses can exceed the interest incomes in general (DDV 2017: 62).

Another risk of an RCB is that investors can not benefit from price increases of the underlying above the exercise price, compared to the direct investment, which leads to foregone profits. Likewise, the investor is not entitled to receive dividends in case of holding an RCB and has to forgo potential dividend payments of the underlying, which arise within the investment period (DDV 2017: 62).

This structured product is particularly suitable for security-conscious investors, who want to reduce the risk of buying a stock and anticipate a sideways-moving, slightly declining or slightly rising market development (Löhr and Cremers 2007: 37; Wilkens and Scholz 2000: 173). Because of the high interest rate, investing in an RCB is less risky than a direct investment in the underlying asset, but usually riskier than an ordinary bond investment (Szymanowska et al. 2009: 3). The variation of the underlying defines the riskiness of the RCB and the level of the coupon rate.

Investors can express their individual risk appetite and their return expectation by choosing a customized RCB with regard to the position of the exercise price in relation to the current share price and the running time. The lower the exercise price of the RCB, the smaller is the interest coupon and the maximal return. The return potential generally increases with the duration time.

3.1.2. Evaluation by Duplication

The final payout at the expiry date depends on the evolution of the underlying and can be expressed as

$$RCB_T = \begin{cases} NV, & \text{if } S_T \geq K \\ aS_T, & \text{if } S_T < K \end{cases} \tag{55}$$

For the valuation and the simulation of potential future returns of an RCB investment, it is necessary to decompose the payout profile into separate elements. Due to the fixed term of this product, the embedded derivatives are of European type.

Based on the implied right of decision of the issuer regarding the repayment form, the cash flow profile of an RCB following Löhr and Cremers (2007: 38) can be represented as

$$RCB_T = min(NV, aS_T) \tag{56}$$
$$= -max(-NV, -aS_T) \tag{57}$$
$$= NV - max(0, NV - aS_T) \tag{58}$$
$$= NV - a \times max\left(\frac{NV}{a} - S_T, 0\right). \tag{59}$$

The payoff profile can be duplicated by holding a zero-coupon bond, which repays the par value at maturity date T, and selling a put options with strike price $\frac{NV}{a}$. Therefore, the payout structure of an RCB can be disassembled in one bond and in one derivative component. It should be noted that the option premium at time $t = 0$ and the risk-free interest payments form the coupon payment. Thus, the above average coupon payment is financed by the received option premium of the sold option.

Through the put-call parity, a second construction methodology is possible. Hence, the payout profile also can be decomposed as

$$RCB_T = min(NV, aS_T) \tag{60}$$
$$= aS_T + min(NV - aS_T, 0) \tag{61}$$
$$= aS_T - max(aS_T - NV, 0) \tag{62}$$
$$= aS_T - a \times max\left(S_T - \frac{NV}{a}, 0\right). \tag{63}$$

The alternative construction yields a combination of a stocks and a sold calls with a strike price of $K = \frac{NV}{a}$.

According to DDV (2017: 66), the relative size of the interest rate depends largely on the expected volatility of the underlying. In the option market, higher premiums are paid for strong price fluctuations as compared to low price fluctuations. Since an RCB guarantees a synthetic put option, a highly volatile underlying asset results in higher offered coupon rates.

3.2. Bonus Cap Certificate

In general, bonus certificates are a popular structured product for retail investors that incorporate an exotic option, namely a path-dependent barrier option (Baule and Tallau 2011: 1).

Structured financial retail products, especially with path-dependent structures, record a growing success since the late 1990s. Though many of those have suffered from the financial crisis, bonus certificates are still being issued and demanded. Mostly they are

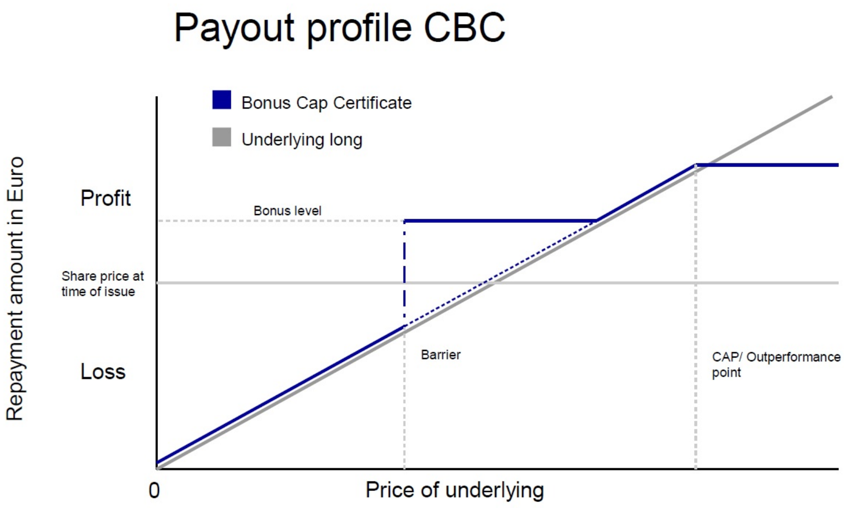

Figure 5: Payout Profile of Bonus Cap Certificate. Source: Own diagram based on DDV (2017: 41) and SIX (2018).

sold with an upper cap, leading to the name bonus cap certificates or likewise capped bonus certificate (CBC) (Hernández et al. 2008: 5).

3.2.1. Structure

An investment in a bonus cap certificate provides an increased repayment in the amount of the bonus level (BL) at the maturity date if the price of the underlying never touches or falls below a predefined price barrier (B) during the term. The *CAP* value determines the ceiling of the payout. At the issue date (t_0), the barrier is set below and the bonus level above the current share price.

Assuming the barrier has never been reached or breached, the CBC will provide the bonus level as a minimum return and the *CAP* as the potential maximum return (Löhr and Cremers 2007: 51). This is shown in Figure 5 with two horizontal blue lines. Between the bonus level and the *CAP*, the investor can participate directly in changes in the share prices of the underlying asset, whereas the magnitude depends on the design of the CBC and its parameters. In practice, these two prices are frequently on the same level, such that the bonus level also depicts the maximum return of the CBC.

As soon as the barrier is crossed or touched once in the investment period, the CBC loses its bonus function and it then resembles a direct investment in the underlying with an upper return limit (CAP). Consequently, the investor will participate completely in potential losses without downward protection and a total loss is possible.

In principle, after the barrier has been hit, the CBC becomes a regular tracker certificate with a *CAP* (Meyer 2013: 18). The payoff profile of a tracker certificate has the same structure as a direct investment in the underlying asset, with the difference that no dividend payments are made. According to Meyer (2013: 18), tracker certificates are optimal for underlying assets which are not appropriate for direct investments, such as indices like the Euro Stoxx 50 or the DAX.

The holder of a CBC has no claim on dividend payments of the underlying, since the issuer withholds these funds to finance the bonus opportunity (Löhr and Cremers 2007: 52; DDV 2017: 38; Meyer 2013: 18). In exchange, the holder achieves a positive return if the underlying only moves sideways or even slightly downwards.

The inclusion of an upper limit for the profit (*CAP*) enables the issuer to define a low-lying barrier in order to reduce the risk of a barrier hit (DDV 2017: 45). Hence, a product for more risk-averse investors can be created. In addition, the bonus mechanism for underlying assets which generate no dividend payments can only be financed through a *CAP*.

The main risk of an investment in a CBC manifests itself through sharply falling prices of the underlying and the accompanied expiration of the claim for the bonus payment. In accordance with Löhr and Cremers (2007: 52), the protection against possible losses and the probability, that the investor gets at least the bonus payment is higher, the greater the distance to the barrier. The distance from the price threshold can be calculated through

$$\text{distance to barrier (in \%)} = \left(1 - \frac{\text{barrier}}{\text{current price of underlying}}\right) \times 100. \tag{64}$$

It is an important variable for the estimation of the incorporated investment risk.

Another important figure is the size of the bonus return, which can be calculated according to DDV (2017: 39) by the formula

$$\text{bonus return} = \left(\frac{\text{bonus level} - \text{purchase price}}{\text{purchase price}}\right) \times \frac{365}{\text{investment period in days}}. \tag{65}$$

Both key figures together are very suitable to evaluate the risks and the chances of a product, since they move in opposite directions.

In principle, a CBC with a low barrier is less risky than a product with a higher barrier, however, it is accompanied by a lower bonus payment (DDV 2017: 44). By contrast, the higher the bonus level is selected, the larger is the bonus return and the smaller the distance to the safety threshold. The earning potential of a CBC generally increases with the term, but also the probability of a barrier hit rises. The investor has to weigh up the trade-off between both factors.

3.2.2. Evaluation by Duplication

The final payout of a CBC at the expiry date depends on the evolution of the underlying over the whole period and can be expressed as

$$CBC_T = \begin{cases} CAP, & \text{if } S_T \geq CAP \\ S_T, & \text{if } BL \leq S_T < CAP \\ BL, & \text{if } S_T \leq BL \ \wedge \ \forall t : S_t > B,\ t \in [0,T] \\ S_T, & \text{if } S_T \leq BL \ \wedge \ \exists t : S_t \leq B,\ t \in [0,T] \end{cases} \tag{66}$$

For the valuation and the simulation of potential future returns of a CBC investment, it is necessary to decompose the payout profile into separate elements, just the same procedure as for the RCB. Due to the fixed term of this product and the fixed maturity date, the embedded derivatives are of European type (Löhr and Cremers 2007: 52).

If the barrier has been violated at least once during the period, the bonus mechanism of the CBC dissipates and the investor receives the payoff profile of the underlying asset with the *CAP* as maximum profit.

The cash flow profile of a CBC at maturity time T after decomposing in separate elements can be represented as

$$\begin{aligned} CBC_T &= max(S_T, BL)\, \mathbb{1}_{\forall t: S_t > B} + S_T\, \mathbb{1}_{\exists t: S_t \leq B} + min(0, CAP - S_T) && (67) \\ &= max(S_T, BL)\, \mathbb{1}_{\forall t: S_t > B} + S_T\, \mathbb{1}_{\exists t: S_t \leq B} + S_T\, \mathbb{1}_{\forall t: S_t > B} - S_T\, \mathbb{1}_{\forall t: S_t > B} && (68) \\ &\quad + min(0, CAP - S_T) \\ &= \underbrace{max(BL - S_T, 0)\, \mathbb{1}_{\forall t: S_t > B}}_{\text{down-and-out put}} + \underbrace{max(S_T, 0)}_{\text{zero-strike call}} - \underbrace{max(S_T - CAP, 0)}_{\text{short call}}. && (69) \end{aligned}$$

Consequently, the payoff profile of a CBC can be duplicated by holding a long position in a down-and-out put option, a long position in a zero-strike call and a short call on the underlying security (Baule and Tallau 2011: 5; Löhr and Cremers 2007: 55). A zero-strike call represents exactly the value of the underlying at maturity through the strike at zero, if the underlying asset pays no dividends within the term of the certificate. The *CAP* mechanism can be implemented by selling a call option with a strike price in the amount of the *CAP* (DDV 2017: 45).

The exercise price of the down-and-out put complies with the bonus level of the certificate, and its barrier defines the barrier of the CBC (DDV 2017: 42). If the investor demands a higher bonus level, she has to pay a higher option premium for the down-and-out put at the issue date, therefore reducing the risk buffer of the total investment. Whenever the underlying asset trades above the bonus level at the valuation date, the down-and-out put option has no intrinsic value, so that solely the underlying's share price and the *CAP* are relevant for the redemption amount of the CBC. This amount is represented by the value of the zero-strike call and short call in equation (69).

Provided that the barrier was violated once during the observation period, the down-and-out put expires immediately and gets worthless. In that case, the redemption amount depends likewise on the zero-strike call and the short call. Given that the share price of the underlying trades above the *CAP* level, the short call will be exercised.

4. Performance of Different Investment Strategies

Certificates have complex structures with different yield potential and various risk profiles. Particularly relevant for the investor is the future performance of the structured product and to what extent it fits her market expectation.

In this chapter, we investigate the performance of reverse convertible bonds and capped bonus certificates for different risk profiles under various market scenarios in comparison to benchmark investments. The objective of this study is to analyze how these two certificate types possibly perform among the changing market developments, considering the different risk appetites, and which investment generates the greatest possible return under the risk restrictions.

According to Hommel and Schiereck (2004: 4) and DDV (2013: 1), certificates offer profit opportunities in all market situations and suitable products for every risk propensity. By means of a Monte Carlo simulation and a performance analysis of RCBs and CBCs, it is possible to evaluate this statement and demystify the potential return distribution of such complex products.

The analysis is based on the same assumptions as the study by Dörer et al. (2017). Therefore, dividend payments and costs for the capital investment are excluded. A strong creditworthiness of the certificate issuer is assumed and the risk figures are determined in conformity with the EU-PRIIPs-Regulation (2017), whereby PRIIPs stands for 'packaged retail and insurance-based investment products'. Furthermore, it is assumed that the holding period of each investment is one year with 256 trading days. The Euro Stoxx 50 index is taken as the underlying asset.

As a first step, the risk classes of the benchmark investments are determined in accordance with the EU-PRIIPs-Regulation (2017). In this document, a detailed evaluation of a PRIIP's risk is outlined.

The benchmark investments are characterized by a combination of a risky investment in a tracker certificate based on the Euro Stoxx 50 index (direct investment in the underlying) and a risk-free bank deposit. The proportion between the risky and the risk-free investment identifies the overall risk figure (market risk measure class from EU-PRIIPs-Regulation (2017)) of each benchmark investment, which is determined by calculating the VaR and subsequently the VaR-equivalent volatility (VEV). Thus, the performance analysis results of a benchmark investment can be compared to an optional certificate investment that is marked with the same risk class. The risk classes 3, 4, and 5 are particularly relevant for the study since they correspond to the most commonly issued certificate investments under consideration.

For the determination of the VEV of the benchmark investments, historical data of the Euro Stoxx 50 index (ES50) from the last three years (2015-2017) is taken from Ariva.de (2018). The risk-free investment represents a bank deposit account with an interest rate of 0.2%, which was the average interest rate in 2017 for a bank deposit in Germany (Statista 2018).

The market risk classes for the certificates are taken from the key information documents (Vontobel 2018a; Vontobel 2018b; Vontobel 2018c; Vontobel 2018d). For the reverse convertible bonds, the most important risk classes are 3 and 4. The most frequently issued capped bonus certificates have a risk class of 4 or 5.

In a second step, 20.000 price paths of the ES50 are simulated via a Monte Carlo simulation, with the underlying assumption that the index follows a GBM. For each risk class, four varying market scenarios are considered with different simulation parameters regarding the expected value and the standard deviation of the underlying. Therefore, the distribution of the ES50 value at maturity is estimated four times. The corresponding prices are used to calculate potential yearly returns representing the estimated future distribution of the return. As a result, for each risk class four estimated distributions of the yearly return are generated.

For each estimated distribution, the expected return and the standard deviation are calculated. Furthermore, the median, the probability of success, the VaR and the Sharpe Ratio are determined to analyze the potential future performance of every benchmark investment.

In case of the alternative certificate investments, the future yearly return distributions are likewise estimated by Monte Carlo simulations of the underlying and a subsequent calculation of the payout at maturity considering the evaluation by duplication method from chapter 3. The returns are the ratio of the difference between the payout values and the initial capital spendings to the initial capital spendings.

Likewise, for each certificate investment four feasible market scenarios are taken into account and via the estimated future distribution of the returns the expected returns, the standard deviations, the medians, the VaRs, the probabilities of success and the Sharpe Ratios are estimated.

4.1. Risk Profiles

There are three types of investors with regard to the risk aversion.

The first risk profile depicts a relatively conservative investor with a risk category of 3 in accordance with the EU-PRIIPs-Regulation (2017). In case of the benchmark investment, this type of investor allocates merely 50% of her capital in the risky asset. The other half she invests into the risk-free bank deposit. For the RCB investment, such an investor chooses a relatively low coupon product. In general, this risk class is characterized by a rather risk-averse and conservative behavior. The risk of loss is low.

The second risk profile describes a balanced investor with a risk category of 4. For the benchmark investment, she invests 75% of her funds in the risky asset and 25% in the deposit account. With regard to the RCB investment, the investor chooses a higher coupon product than the conservative investor, therefore takes higher risks. For a CBC she chooses a product with a bonus level relatively close to the initial reference price. Such an investor is rather risk-seeking, but the risk of loss is balanced or rather intermediate.

The last risk profile characterizes a profit-oriented investor with a risk category of 5. She invests 100% of her funds into the risky asset in case of the benchmark investment. For the CBC investment, she picks a product with high bonus level or high barrier. This type of investor is strongly risk-seeking and accepts a large risk of loss.

The risk categories for the certificates pursuant to the EU-PRIIPs-Regulation (2017) are extracted from the key information documents. For the RCBs they are outlined in Vontobel (2018a) and Vontobel (2018b), respectively, for the CBCs in Vontobel (2018c) and Vontobel (2018d).

The assignment of the risk categories to the benchmark investments is not that straightforward. At this point, only the main steps are exemplified. The methodology of the market risk evaluation of a PRIIP is explained in the EU-PRIIPs-Regulation (2017) in detail.

As fundamental data set, the ES50 index values of the past three years (2015-2017) are extracted from Ariva.de (2018), such that the daily return distribution can be generated. At first, the 2.75%-VaR in return-space has to be calculated via the Cornish-Fisher expansion as follows (EU-PRIIPs-Regulation 2017: 16):

$$VaR_{\text{return-space}} = \sigma\sqrt{N}\left(-1.96 + 0.474\frac{\mu_1}{\sqrt{N}} - 0.0687\frac{\mu_2}{N} + 0.146\frac{\mu_1^2}{N}\right) - 0.5\sigma^2 N \quad (70)$$

$N =$ number of trading periods in the recommended holding period

$\sigma =$ volatility from the return distribution

$\mu_1 =$ skew from the return distribution

$\mu_2 =$ excess kurtosis from the return distribution

With the resulting VaR, the VEV can then be calculated by

$$VEV = \left(\sqrt{3.842 - 2VaR_{\text{return-space}}} - 1.96\right) \frac{1}{\sqrt{T}}, \tag{71}$$

where T is the length of the recommended holding period in years (it is assumed that $T = 1$). An example calculation for the ES50 is provided in ESAs-Joint-Committee (2017: 8).

With the help of a prescribed table, the VEV values can be assigned to specific market risk categories, ranging from 1 to 7. Therefore, for every combination of the tracker certificate and the bank deposit (benchmark investments), an individual risk value is specified. As mentioned before, exclusively relevant for this study are the risk categories 3-5.

4.2. Market Scenarios

Besides the different degrees of risk aversion, the potential market scenarios in which an investment in an RCB or in a CBC performs better/worse than a benchmark investment constitute an essential factor within the performance analysis.

Obviously, there are countless possible future market scenarios of the ES50, depending on the evolution of the economic and political situation. In this study, four conceivable market scenarios are considered in order to represent the future performance of the certificates as precise as possible with the preservation of good manageability. The four selected market scenarios of the ES50 are distinguished by different parameters for the expected return and the volatility in the Monte Carlo simulation. For each market scenario, a separate Monte Carlo simulation of the underlying is conducted.

In an optimistic scenario, the ES50 evolves with an expected return of 6.5% and volatility of 7.5%. Hence, it is characterized by a high expected return and low volatility.

The moderate scenario is based on the ES50 parameters of the year 2017, therefore the index evolves with an expected return of 5.76% and a volatility of 10.29%. It can be seen as a moderate development.

In a pessimistic scenario, the underlying develops with an expected return of 4.75% and a volatility of 15%, and thus with a lower expected return and a higher volatility than in 2017.

The last scenario is characterized by a turbulent market situation with an expected return of 3.75% and a volatility of 20% of the ES50, so that large fluctuations can be expected. Therefore, it depicts a stress scenario with a high volatility and a low expected return of the underlying.

4.3. Benchmark Versus Reverse Convertible Bond

The first performance analysis concerns the investment in two different reverse convertible bonds in comparison to the benchmark investments with risk categories 3 and 4. Overall, the simulation shows that an investment in an RCB results in positive returns with a high probability regardless of the risk profile and the considered market scenario.

On the one hand, the RCB with risk category 3 ensures a very high probability of success and a lower loss potential, in comparison to the RCB with risk category 4 in every scenario. On the other hand, the RCB with risk profile 4 guarantees a higher expected return, which the investor must buy for a higher risk of loss.

In the optimistic, moderate and pessimistic market scenarios, both reverse convertible bonds perform better than the benchmark investments, indicated by higher success probabilities, median values, and Sharpe Ratios. In the stress scenario, the benchmark investments perform slightly better, because of the higher expected returns, Sharpe Ratios and 1%-VaR values. This result is due to the upper limits of the RCBs returns, whereas an benchmark investor can participate endless in increasing index values.

However, it must be taken into account that even in the stress scenario, the probabilities of success and the median values are significantly larger with RCBs as compared to the benchmark investments. Therefore, an investment in an RCB is optimal for security-oriented investors who prefer a high success probability over high expected returns. Even if it comes to a negative market development, the RCB investment achieves a positive return with a high probability.

4.3.1. Optimistic Scenario

Optimistic scenario				
Mu = 6.5% and sigma = 7.5%				
	PRIIP 3		PRIIP 4	
	Benchmark (50%*rf+50%*tracker)	RCB (4,50%)	Benchmark (25%*rf+75%*tracker)	RCB (8%)
Mean	3.45%	4.37%	5.08%	6.55%
Median	3.32%	4.50%	4.88%	8.00%
Standard deviation	3.99%	0.80%	5.99%	2.93%
Probability of success	80.00%	99.05%	79.51%	94.45%
1%-VaR	-5.05%	0.14%	-7.68%	-4.53%
Sharpe Ratio	0.81	5.18	0.81	2.17
95% confidence interval	[-3.86%; 11.75%]	[2.69%; 4.50%]	[-5.88%; 17.53%]	[-2.20%; 8.00%]

Table 2: Optimistic Scenario RCB. Source: Own research.

The results of the performance analysis of the RCBs in the optimistic scenario are shown in Table 2.

In the optimistic scenario, the expected return of the RCB after one year is almost one percentage point higher for the conservative investor in comparison to the benchmark investment. The probability of success has a value of 99% in that case, whereas the benchmark is quoted at a success rate of 80%. With a value of 5.18, the Sharpe Ratio is more than six times larger, indicating a higher risk-adjusted return.

For the balanced investor, the expected RCB return is even two percentage points higher. In that risk category, the Sharpe Ratio is more than twice as large as the value for the benchmark.

4.3.2. Moderate Scenario

Moderate scenario				
Mu = 5.757235% and sigma = 10.28663% (based on parameters of 2017)				
	PRIIP 3		PRIIP 4	
	Benchmark (50%*rf+50%*tracker)	RCB (4,50%)	Benchmark (25%*rf+75%*tracker)	RCB (8%)
Mean	3.06%	3.87%	4.49%	5.27%
Median	2.80%	4.50%	4.10%	8.00%
Standard deviation	5.45%	2.16%	8.17%	4.67%
Probability of success	70.12%	94.11%	69.71%	85.68%
1%-VaR	-8.22%	-6.62%	-12.44%	-10.72%
Sharpe Ratio	0.52	1.70	0.52	1.09
95% confidence interval	[-6.69%;14.58%]	[-3.36%; 4.50%]	[-10.14%; 21.77%]	[-7.73%; 8.00%]

Table 3: Moderate Scenario RCB. Source: Own research.

In Table 3, the results for the moderate scenario are depicted.

The conservative investor can anticipate a 0.81 percentage points higher return for the reverse convertible investment with less volatility in the estimated return distribution if the ES50 moves like in 2017. In addition, it promises a 25 percentage points higher success probability and an about three times larger Sharpe Ratio, so that the RCB investment is to be preferred.

The balanced investor can as well benefit from a larger expected return with less volatility and from a higher probability of success, although the differences to the figures of the benchmark are somewhat smaller in this risk category.

4.3.3. Pessimistic Scenario

Pessimistic scenario				
mu = 4.75% and sigma = 15%				
	PRIIP 3		PRIIP 4	
	Benchmark (50%*rf+50%*tracker)	RCB (4,50%)	Benchmark (25%*rf+75%*tracker)	RCB (8%)
Mean	2.52%	2.39%	3.68%	3.02%
Median	1.97%	4.50%	2.86%	8.00%
Standard deviation	7.88%	4.82%	11.83%	7.40%
Probability of success	60.34%	83.28%	60.00%	73.41%
1%-VaR	-13.06%	-16.94%	-19.69%	-20.16%
Sharpe Ratio	0.29	0.45	0.29	0.38
95% confidence interval	[-11.07%; 19.71%]	[-12.70%; 4.50%]	[-16.71%; 29.46%]	[-16.28%; 8.00%]

Table 4: Pessimistic Scenario RCB. Source: Own research.

The results for the pessimistic scenario are illustrated in Table 4.

In the pessimistic scenario, the conservative investor who invests in an RCB has a performance advantage in comparison to an RCB investor with risk category 4. Even though the expected returns for both risk profiles are greater for the benchmark investments, it has to be taken into account that the mean reacts strongly to the negative outliers.

The median is more robust and meaningful at this point. In the case of the RCB investments, the median values equal the predefined coupon rates of each product, which outperform the values of the benchmark investments. Furthermore, the probabilities of success and the Sharpe Ratios of the RCBs for both risk categories are larger.

4.3.4. Stress Scenario

Stress scenario				
Mu = 3.75% and sigma = 20%				
	PRIIP 3		PRIIP 4	
	Benchmark (50%*rf+50%*tracker)	RCB (4,50%)	Benchmark (25%*rf+75%*tracker)	RCB (8%)
Mean	2.00%	0.39%	2.89%	0.59%
Median	1.02%	4.50%	1.42%	7.30%
Standard deviation	10.45%	7.66%	15.68%	10.08%
Probability of success	54.02%	73.68%	53.73%	64.82%
1%-VaR	-17.64%	-26.70%	-26.56%	-29.08%
Sharpe Ratio	0.17	0.02	0.17	0.04
95% confidence interval	[-15.30%; 25.46%]	[-21.71%; 4.50%]	[-23.05%; 38.09%]	[-24.51%; 8.00%]

Table 5: Stress Scenario RCB. Source: Own research.

The results of the stress scenario from Table 5 show a slightly better performance of the benchmark investments due to the higher volatility of the underlying. The expected returns of the benchmark investments are larger and the loss amounts in 1% of the cases are smaller.

Nevertheless, the RCB investments realize higher median values and higher success probabilities for both risk categories. It is left to the investor whether she sets her priority to a high expected value or a high probability of success.

4.4. Benchmark Versus Bonus Cap Certificate

The second performance analysis concerns the investment in two different capped bonus certificates in comparison to the benchmark investments with risk categories 4 and 5. The simulation shows that an investment in a CBC results in positive returns with a high probability as well, regardless of the risk profile and the considered market scenario.

The profit-oriented CBC investor has a higher probability of success in each market scenario in comparison to the balanced CBC investor. However, she takes a higher risk of loss, because of the higher purchasing price of the CBC. Especially when it comes to bad market developments, she has to anticipated larger losses.

As well as with the RCB investments, the capped bonus certificates perform better than the benchmark investments in the optimistic, the moderate and the pessimistic market scenarios, indicated by higher success probabilities, median values, and Sharpe Ratios. The expected returns and the 1%-VaR values for both CBCs, however, lie below the figures of the benchmark investments in the pessimistic and the stress scenarios.

In the stress scenario, the benchmark investments perform slightly better, because of the higher expected returns, Sharpe Ratios, and 1%-VaR values. As with RCB investments, this result is due to the upper limits of the CBCs returns, whereas a benchmark investor can participate endless in increasing index values. Once again, it must be taken into account that even in the stress scenario, the probabilities of success and the median values are significantly larger of the CBCs than those of the benchmark investments.

4.4.1. Optimistic Scenario

Optimistic scenario				
Mu = 6.5% and sigma = 7.5%				
	PRIIP 4		PRIIP 5	
	Benchmark (25%*rf+75%*tracker)	CBC	Benchmark (100%*tracker)	CBC
Mean	5.08%	8.37%	6.71%	6.83%
Median	4.88%	8.77%	6.44%	6.84%
Standard deviation	5.99%	2.38%	7.99%	0.41%
Probability of success	79.51%	97.41%	79.23%	99.99%
1%-VaR	-7.68%	-6.16%	-10.30%	6.84%
Sharpe Ratio	0.81	3.43	0.81	16.15
95% confidence interval	[-5.88%; 17.53%]	[-0.67%; 8.77%]	[-7.91%; 23.30%]	[6.84%; 6.84%]

Table 6: Optimistic Scenario CBC. Source: Own research.

The simulation results of the CBCs in the optimistic scenario are depicted in Table 5.

A balanced investor can expect a 3.3 percentage points higher return when investing in a capped bonus certificate rather than investing in the benchmark. The probability of success is by almost 18 percentage points higher. In the case of the profit-oriented investor, the success probability is nearly 100%.

4.4.2. Moderate Scenario

Moderate scenario				
Mu = 5.757235% and sigma = 10.28663% (based on parameters of 2017)				
	PRIIP 4		PRIIP 5	
	Benchmark (25%*rf+75%*tracker)	CBC	Benchmark (100%*tracker)	CBC
Mean	4.49%	6.93%	5.91%	6.69%
Median	4.10%	8.77%	5.40%	6.84%
Standard deviation	8.17%	5.09%	10.89%	2.15%
Probability of success	69.71%	89.15%	69.49%	99.54%
1%-VaR	-12.44%	-12.88%	-16.65%	6.84%
Sharpe Ratio	0.52	1.32	0.52	3.02
95% confidence interval	[-10.14%; 21.77%]	[-9.47%; 8.77%]	[-13.59%; 28.95%]	[6.84%; 6.84%]

Table 7: Moderate Scenario CBC. Source: Own research.

The performance results for the moderate scenario from Table 7 show that the expected return of the balanced CBC investor is 2.44 percentage points larger with a lower volatility. The probability of success is 20 percentage points higher than the value of the benchmark asset.

A profit-oriented investor who invests in a CBC can expect a 0.78 percentage points greater return and a 30 percentage points higher success probability in comparison to the benchmark investment. In this case, the Sharpe Ratio is more than five times bigger.

4.4.3. Pessimistic Scenario

Pessimistic scenario				
mu = 4.75% and sigma = 15%				
	PRIIP 4		PRIIP 5	
	Benchmark (25%*rf+75%*tracker)	CBC	Benchmark (100%*tracker)	CBC
Mean	3.68%	3.76%	4.84%	4.97%
Median	2.86%	8.77%	3.74%	6.84%
Standard deviation	11.83%	8.59%	15.77%	7.58%
Probability of success	60.00%	74.65%	59.85%	94.04%
1%-VaR	-19.69%	-23.13%	-26.33%	-29.78%
Sharpe Ratio	0.3	0.42	0.3	0.63
95% confidence interval	[-16.71%; 29.46%]	[-18.69%; 8.77%]	[-22.35%; 39.21%]	[-25.73%; 6.84%]

Table 8: Pessimistic Scenario CBC. Source: Own research.

Table 8 depicts the simulation results for the pessimistic scenario.

The pessimistic scenario for the balanced CBC investor is characterized by a slightly higher expected return of 0.08 percentage points. However, the median is 5.91 percentage points larger with a probability of success of nearly 75%. The profit-oriented investor has a 94% chance of success. The loss amount in 1% of the cases is slightly higher for both risk classes when investing in the certificate.

4.4.4. Stress Scenario

Stress scenario				
Mu = 3.75% and sigma = 20%				
	PRIIP 4		PRIIP 5	
	Benchmark (25%*rf+75%*tracker)	CBC	Benchmark (100%*tracker)	CBC
Mean	2.89%	0.66%	3.79%	1.06%
Median	1.42%	8.77%	1.83%	6.84%
Standard deviation	15.68%	11.51%	20.90%	12.82%
Probability of success	53.73%	64.74%	53.53%	81.89%
1%-VaR	-26.56%	-32.80%	-35.48%	-38.61%
Sharpe Ratio	0.17	0.04	0.17	0.07
95% confidence interval	[-23.05%; 38.09%]	[-27.58%, 8.77%]	[-30.80%; 50.72%]	[-33.85%; 6.84%]

Table 9: Stress Scenario CBC. Source: Own research.

The findings for the stress scenario are outlined in Table 9.

In the stress scenario with a high volatility of the underlying, the expected return of the benchmark asset is more than 2 percentage points higher for both risk classes. The 1%-VaR values are significantly larger for the benchmark investments.

However, the median values are larger for the alternative capped bonus certificate investments. The investors of both categories can also benefit from high probabilities of success compared to the benchmark assets.

5. Conclusion

In the continuing low-interest market environment retail investors have to deal with more complex and complicated investment products than bonds or bank deposits if they aspire to obtain interest payments to ensure their old-age insurance. Especially for rather risk-averse and conservative investors, the question arises which investment opportunity in the wide product market is compatible with their risk profile and their expected profit chances.

By means of Monte Carlo simulations and the assumption that the Euro Stoxx 50 index behaves like a geometric Brownian motion, the future performance of representative RCBs and CBCs could be estimated and compared with the performance of benchmark investments of the same PRIIP risk category. In this context, reverse convertible bonds and capped bonus certificates represent an advantageous extension of the investment universe and are therefore promising alternative investments that should not be underestimated.

The performance analysis in this study has shown that the risk profiles of three different retail investors can be depicted by means of RCBs and CBCs, and that even in the low-interest phase, positive returns can be realized with a high probability. In any of the four potential market scenarios, the median values and the success probabilities of the certificate investments are higher in comparison to the figures of the related benchmark investments. Moreover, the Sharpe Ratios take higher values in the optimistic, the moderate and the pessimistic scenarios for every certificate investment under consideration. For a sideways-moving development of the underlying asset with merely small fluctuations, the certificate investments show a significantly better performance.

However, to savor these high probabilities of success, an investor has to accept a lower expected return in case of a potential pessimistic or stress scenario if she allocates her funds in a reverse convertible bond. An investor who invests in a capped bonus certificate has to anticipate a lower expected return solely in the stress scenario, independent of the risk category. The loss amount in 1% of the cases is larger for the certificate investments in the arrival of a stress scenario, due to the upper limits of the returns.

Overall, an investment in an RCB or a CBC is optimal for security-oriented investors who prefer products with easier to predict returns which are rather limited to a specific area. In addition, the investment objective should lie on a high success probability, rather than profuse expected returns.

The main contribution of this thesis is to create transparency regarding the chances and risks of reverse convertible bonds and capped bonus certificates. In conclusion, it can be stated that RCBs and CBCs represent a meaningful investment opportunity. Investors can adjust their individual risk appetite and even get a positive return in a possible stress scenario with a high probability.

References

Ariva.de (2018). *Euro Stoxx 50 Historische Kurse*. URL: https://www.ariva.de/eurostoxx-50-index/historische_kurse (visited on Nov. 10, 2018).

Arnold, L. (1973). *Stochastische Differentialgleichungen: Theorie und Anwendung*. München: Oldenbourg.

Asiri, F. F. (2018). *The Price of Stocks, Geometric Brownian Motion, and Black Scholes Formula*. Major Papers 26. University of Windsor.

Baule, R. and Tallau, C. (2011). "The Pricing of Path-Dependent Structured Financial Retail Products: The Case of Bonus Certificates". *Journal of Derivatives* 18 (4), pp. 1–41.

Black, F. and Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities". *The Journal of Political Economy* 81 (3), pp. 637–654.

Bloss, M. (2017). *Financial Engineering: Strategien, Bewertungen und Risikomanagement*. Berlin/Boston: Walter de Gruyter.

Burth, S., Kraus, T., and Wohlwend, H. (2001). "The pricing of structured products in the Swiss market". *The Journal of Derivatives* 9 (2), pp. 30–40.

Christoffersen, P. F. (2003). *Elements of Financial Risk Management*. San Diego: Academic Press.

DDV (2013). *Daten – Fakten – Argumente: Wozu braucht man eigentlich Zertifikate?* Deutscher Derivate Verband.

DDV (2017). *Kompass Strukturierte Produkte*. Deutscher Derivate Verband.

Dörer, B. et al. (2017). *Der Nutzen strukturierter Wertpapiere für Privatanleger: Discount-Zertifikate als Anlagealternativen im Niedrigzinsumfeld*. Deutscher Derivate Verband. URL: www.derivateverband.de.

ESAs-Joint-Committee (2017). *PRIIPs – Flow diagram for the risk and reward calculations in the PRIIPs KID*. The Joint Committee of the European Supervisory Authorities (EBA, EIOPA, ESMA - ESAs). URL: https://esas-joint-committee.europa.eu/Publications/Technical%20Standards/JC%202017%2049%20(PRIIPs_flow_diagram_risk_reward).pdf.

Glasserman, P. (2004). *Monte Carlo Methods in Financial Engineering*. New York: Springer-Verlag.

Grüne, L. (2011). *Numerische Methoden der Finanzmathematik*. Lecture Notes. Universität Bayreuth.

Guidolin, M. and Pedio, M. (2015). *An Introduction to Structured Financial Instruments*. Lecture Notes. Innocenzo Gasparini Institute for Economic Research.

Hernández, R., Brusa, J., and Liu, P. (2008). "An Economic Analysis of Bonus Certificates—Second - Generation of Structured Products". *Review of Futures Markets* 16 (4), pp. 419–451.

Hommel, U. and Schiereck, D. (2004). *Der Nutzen derivativer Finanzprodukte für Privatanleger*. European Business School.

Hull, J. C. (2018). *Option, Futures, and Other Derivatives*. New York: Pearson Education.

Iacus, S. M. (2011). *Option Pricing and Estimation of Financial Models with R*. Chichester: John Wiley & Sons, Ltd.

Irle, A. (2012). *Finanzmathematik: Die Bewertung von Derivaten*. Wiesbaden: Springer-Verlag.

Kallsen, J. (2015). *Mathematical Finance: An introduction in discrete time*. Lecture Notes. Christian-Albrechts-Universität zu Kiel.

Kallsen, J. (2017a). *Computational Finance*. Lecture Notes. Christian-Albrechts-Universität zu Kiel.

Kallsen, J. (2017b). *Risk Management*. Lecture Notes. Christian-Albrechts-Universität zu Kiel.

Löhr, A. and Cremers, H. (2007). *Deskription und Bewertung strukturierter Produkte unter besonderer Berücksichtigung verschiedener Marktszenarien*. Working Paper No. 82. Frankfurt School of Finance & Management. URL: `www.frankfurt-school.de`.

Merton, R. C. (1973). "Theory of Rational Option Pricing". *The Bell Journal of Economics and Management Science* 4, (1), pp. 141–183.

Meyer, S. (2013). *Trading in Structured Products: Investor Behavior and Pricing Policies*. Dissertation. Karlsruher Institut für Technologie (KIT).

Müller, A. (2016). *An application of the put-call-parity to variance reduced Monte-Carlo option pricing*. Discussion paper No. 495. FernUniversität in Hagen.

EU-PRIIPs-Regulation (2017). *Commission Delegated Regulation (EU) 2017/653 of 8 March 2017 supplementing Regulation (EU) No 1286/2014 of the European Parliament and of the Council on key information documents for packaged retail and insurance-*

based investment products (PRIIPs) by laying down regulatory technical standards with regard to the presentation, content, review and revision of key information documents and the conditions for fulfilling the requirement to provide such documents. URL: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32017R0653 (visited on Dec. 25, 2018).

Schmidt, M. (2014). *Derivate Finanzinstrumente: Eine anwendungsorientierte Einführung*. Stuttgart: Schäffer-Poeschel Verlag.

Sengputa, C. (2004). *Financial Modeling Using Excel and VBA*. Hoboken, New Jersey: John Wiley & Sons, Ltd.

Seydel, R. (2017). *Computational Finance: Einführung in die numerische Berechnung von Finanzderivaten*. Berlin Heidelberg: Springer-Verlag.

Sharpe, W. F. (1966). "Mutual Fund Performance". *The Journal of Business* 39 (1), pp. 119–138.

SIX (2018). *Example of a capped bonus certificate*. URL: https://www.six-structured-products.com/en/know-how/product-know-how/participation-products/bonus-certificate (visited on Dec. 17, 2018).

Statista (2018). *Entwicklung des durchschnittlichen Zinssatzes für Spareinlagen (Sparbuchzinsen) in Deutschland in den Jahren 1975 bis 2017*. URL: https://de.statista.com/statistik/daten/studie/202295/umfrage/entwicklung-des-zinssatzes-fuer-spareinlagen-in-deutschland/ (visited on Nov. 10, 2018).

Szymanowska, M., Horst, J. ter, and Veld, C. (2009). "Reverse Convertible Bonds Analyzed". *Journal of Futures Markets* 29 (10), pp. 895–919.

Vontobel (2018a). *Aktienanleihen auf EURO STOXX 50 Index*. Basisinformationsblatt. ISIN: DE000VL8TBG7 / WKN: VL8TBG. Bank Vontobel Europe AG.

Vontobel (2018b). *Aktienanleihen auf EURO STOXX 50 Index*. Basisinformationsblatt. ISIN: DE000VL8TBK9 / WKN: VL8TBK. Bank Vontobel Europe AG.

Vontobel (2018c). *Bonus Cap Zertifikat auf EURO STOXX 50 Index*. Basisinformationsblatt. ISIN: DE000VA75F16 / WKN: VA75F1. Bank Vontobel Europe AG.

Vontobel (2018d). *Bonus Cap Zertifikat auf EURO STOXX 50 Index*. Basisinformationsblatt. ISIN: DE000VA8SD81 / WKN: VA8SD8. Bank Vontobel Europe AG.

Wilkens, M. and Scholz, H. (2000). "Reverse Convertibles und Discount-Zertifikate - Bewertung, Pricingrisiko und implizite Volatilität". *Finanz Betrieb* 3, pp. 171–179.

Appendix

A. Parameters of Certificates

	RCB (4.50% coupon)	RCB (8% coupon)
ISIN	DE000VL8TBG7	DE000VL8TBK9
WKN	VL8TBG	VL8TBK
Underlying	EURO STOXX 50® Index	EURO STOXX 50® Index
Nominal Amount	EUR 1000	EUR 1000
Interest Rate	4.50% p.a.	8.00% p.a.
Initial Reference Price	EUR 3412.08	EUR 3413.08
Strike	EUR 3200.00	EUR 3500.00
Issue Date	22 February 2018	22 February 2018
Maturity Date (Maturity)	22 March 2019	22 March 2019
PRIIP Risk Indicator	3	4

Table 10: Parameters of the Reverse Convertible Bonds extracted from the Key Information Documents Vontobel (2018a) and Vontobel (2018b).

	CBC (3300 BL)	CBC (3500 BL)
ISIN	DE000VA75F16	DE000VA8SD81
WKN	VA75F1	VA8SD8
Underlying	EURO STOXX 50® Index	EURO STOXX 50® Index
Purchase Price	EUR 30.34	EUR 32.76
Bonus Amount	EUR 33.00	EUR 35.00
Initial Reference Price	EUR 3182.60	EUR 3138.93
Ratio	0.01	0.01
Bonus Level	EUR 3300.00	EUR 3500.00
Barrier	EUR 2850.00	EUR 2450.00
Cap	EUR 3300.00	EUR 3500.00
Issue Date	15 October 2018	25 October 2018
Maturity Date (Maturity)	27 September 2019	27 September 2019
PRIIP Risk Indicator	4	5

Table 11: Parameters of the Capped Bonus Certificates extracted from the Key Information Documents Vontobel (2018c) and Vontobel (2018d).

B. Histograms

B.1. Histograms of Simulated RCB Returns

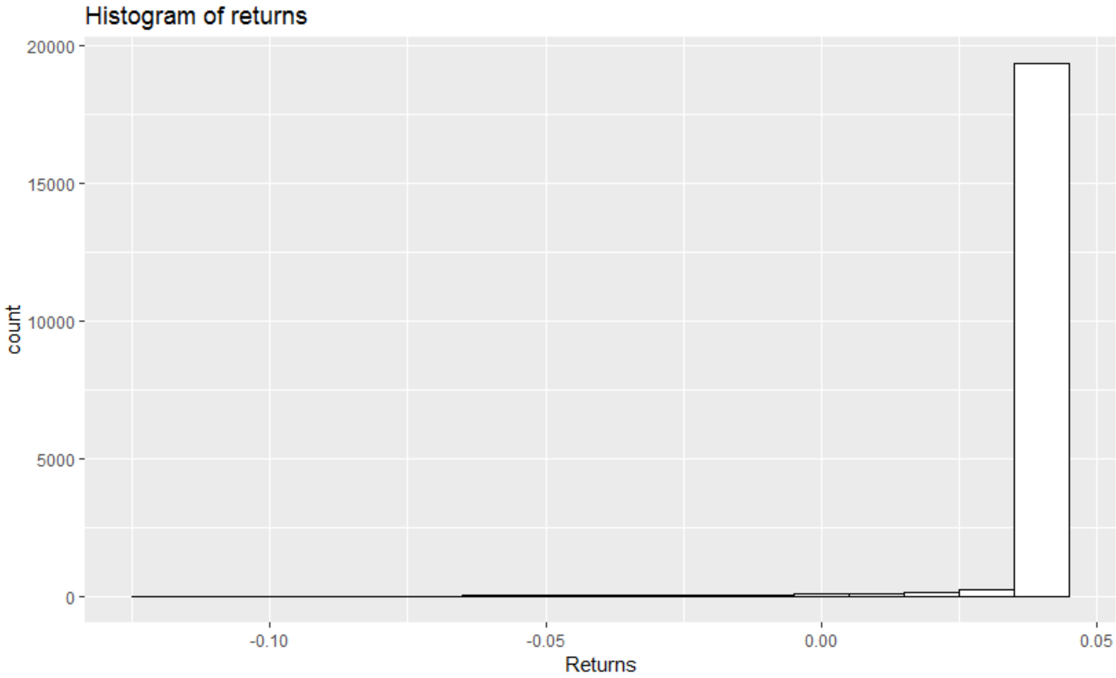

Figure 6: RCB Risk Category 3, Optimistic Scenario. Source: Own diagram.

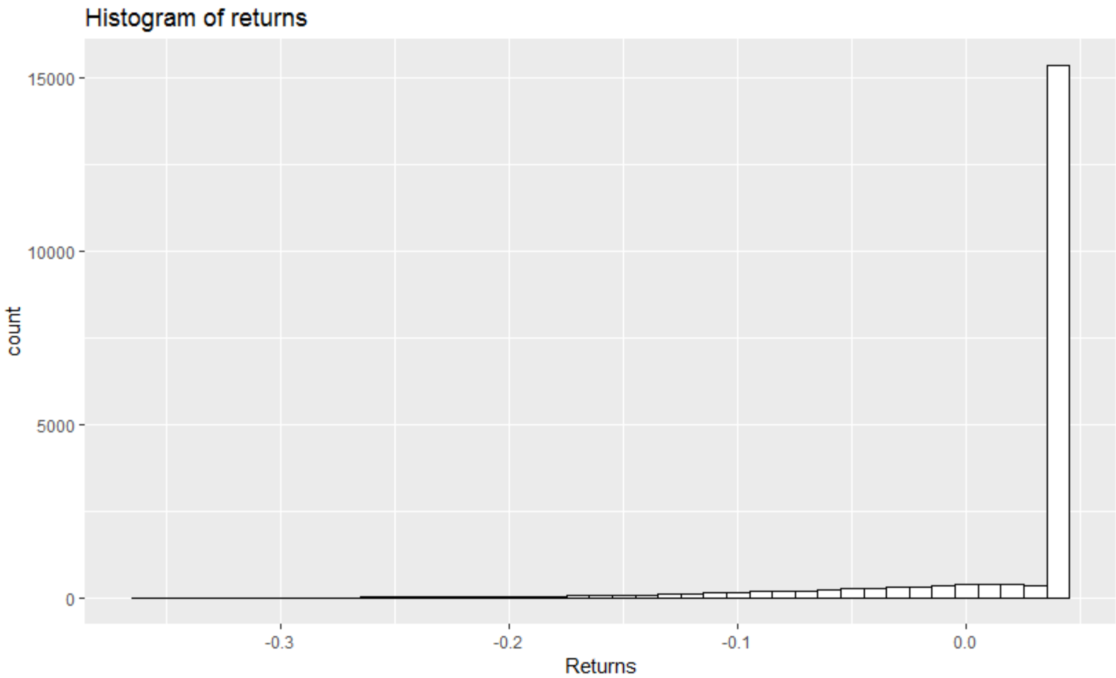

Figure 8: RCB Risk Category 3, Pessimistic Scenario. Source: Own diagram.

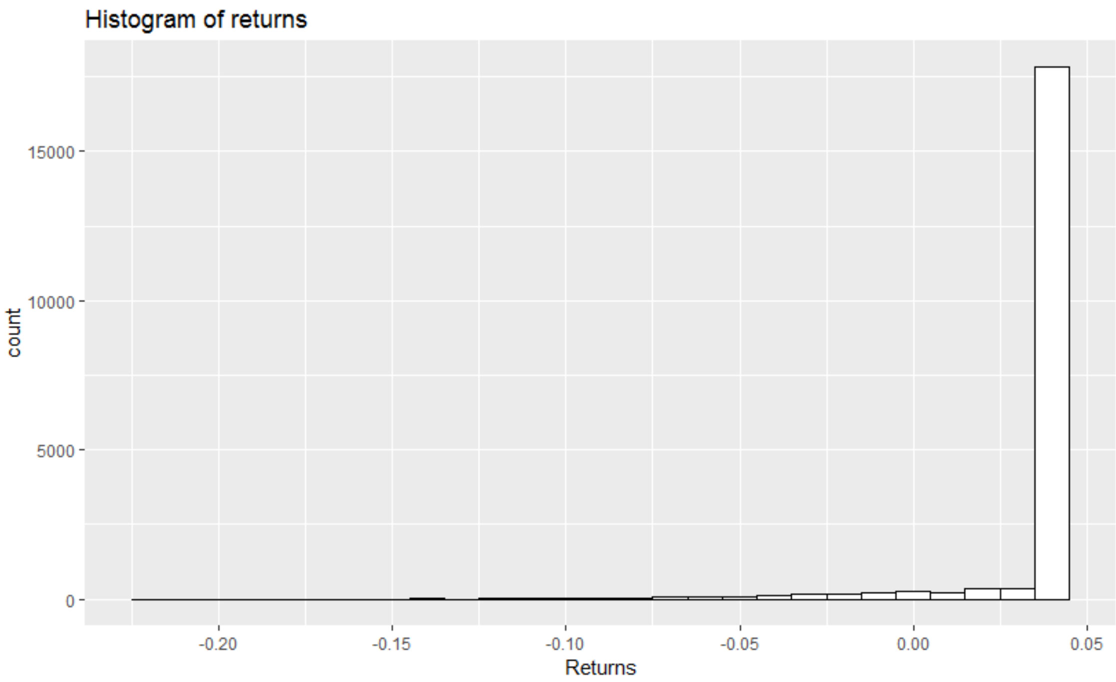

Figure 7: RCB Risk Category 3, Moderate Scenario. Source: Own diagram.

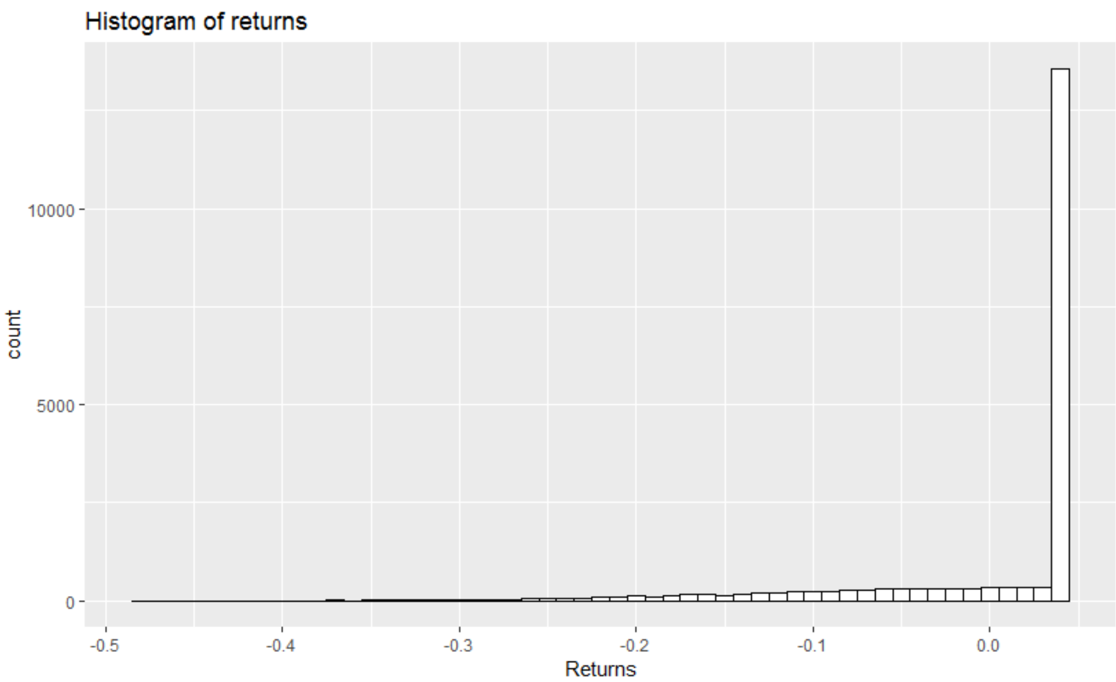

Figure 9: RCB Risk Category 3, Stress Scenario. Source: Own diagram.

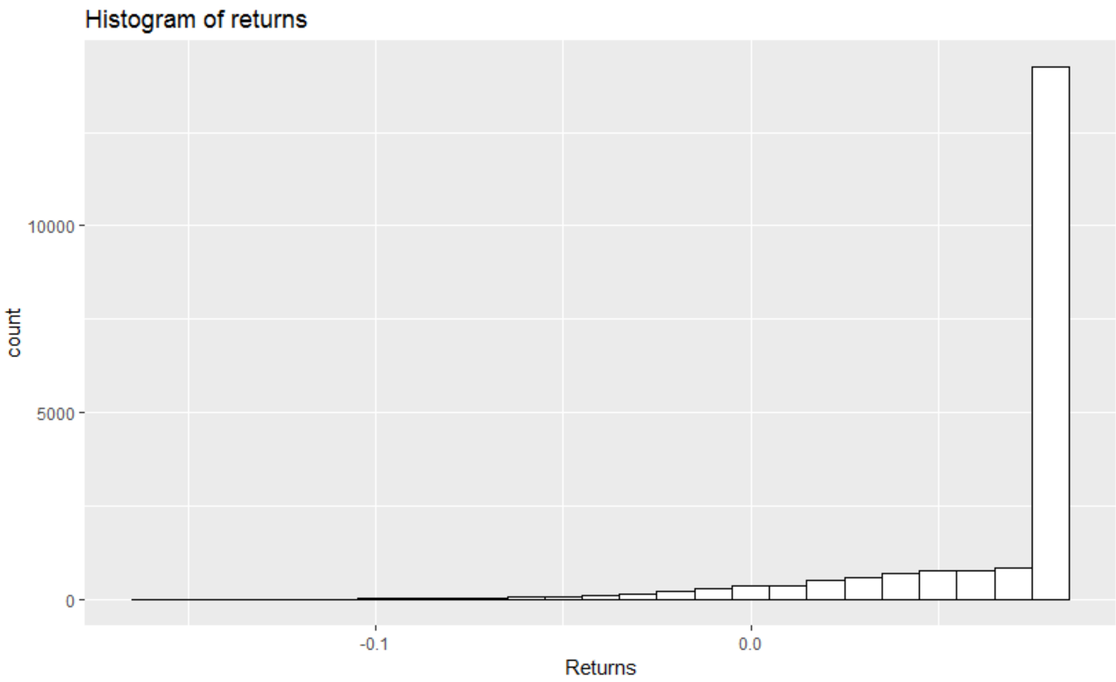

Figure 10: RCB Risk Category 4, Optimistic Scenario. Source: Own diagram.

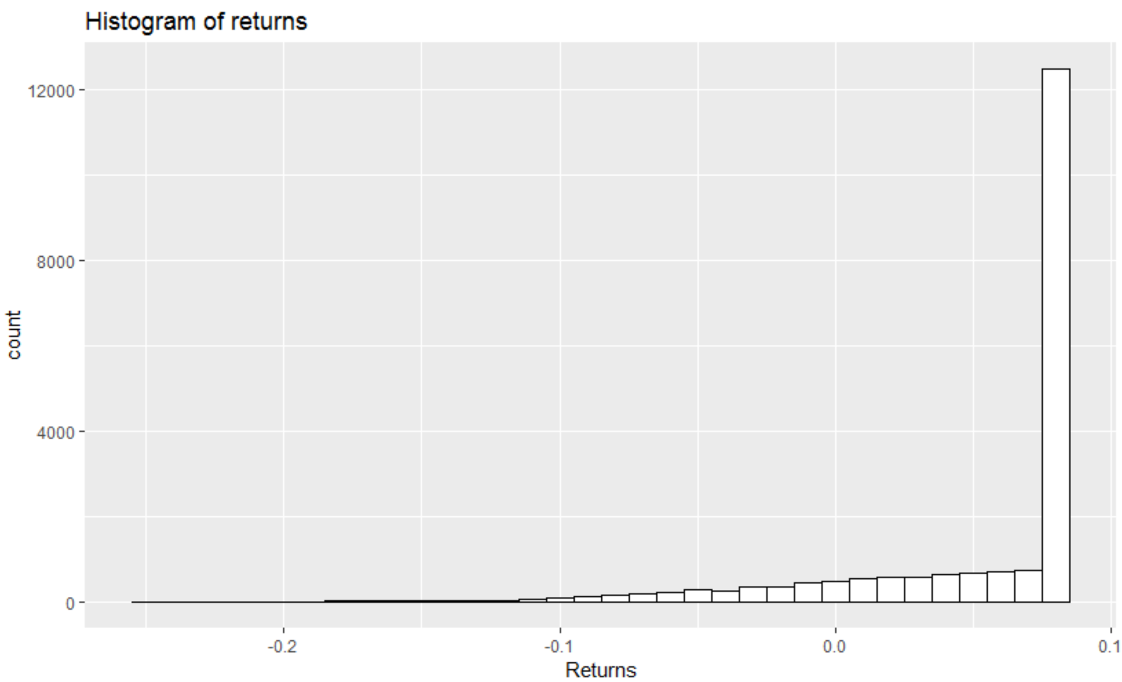

Figure 11: RCB Risk Category 4, Moderate Scenario. Source: Own diagram.

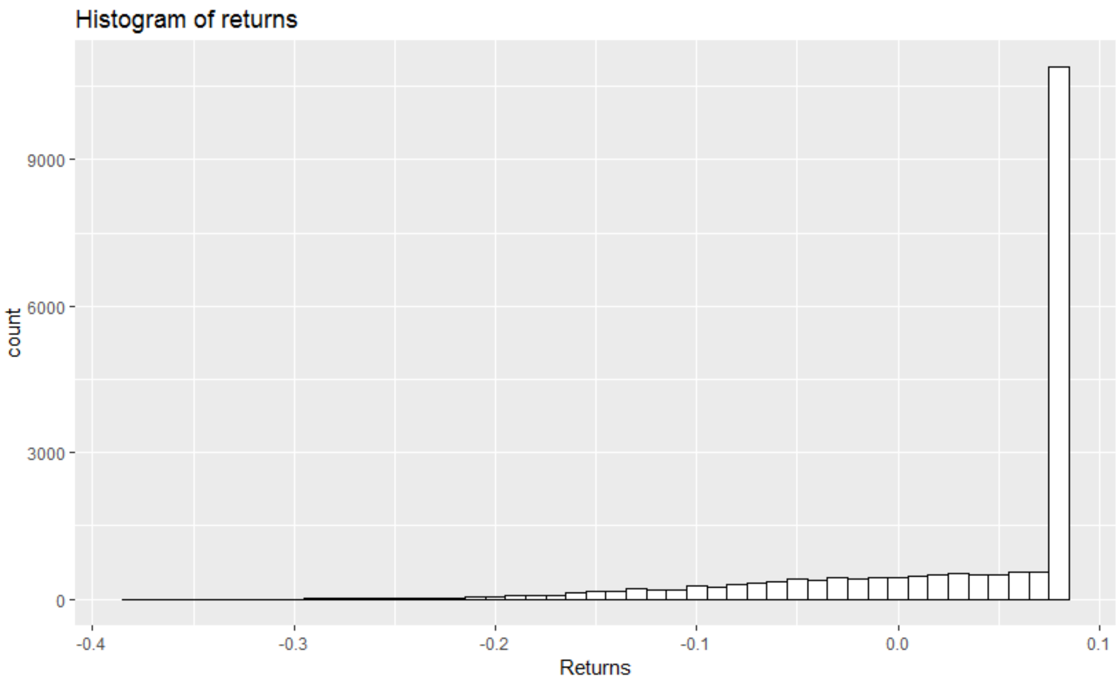

Figure 12: RCB Risk Category 4, Pessimistic Scenario. Source: Own diagram.

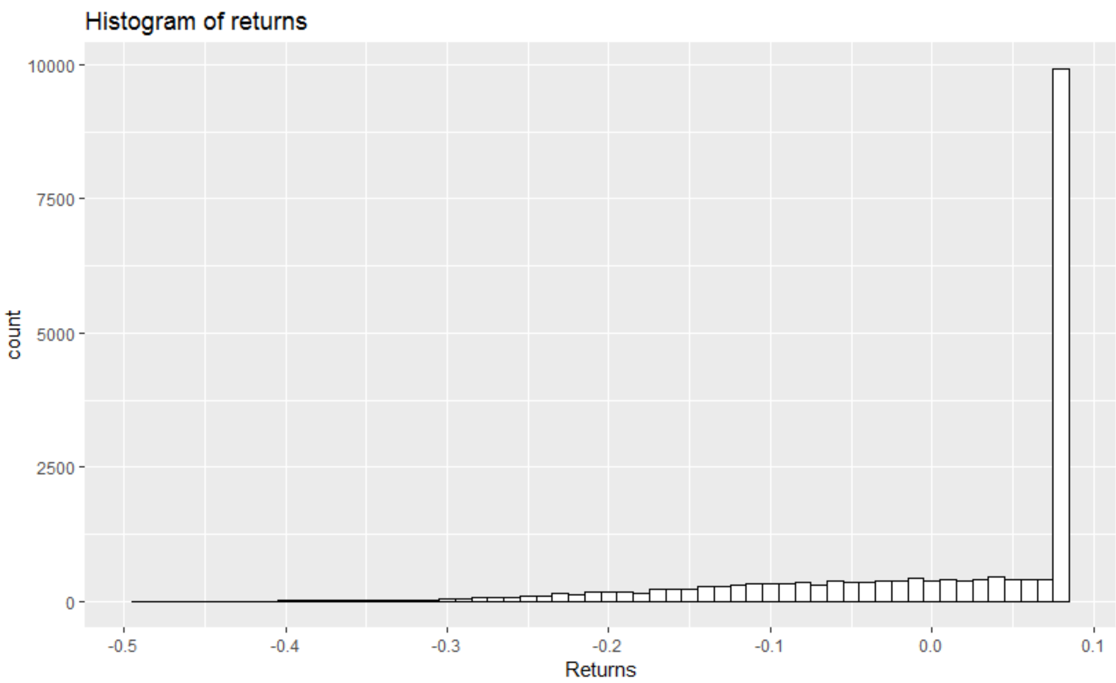

Figure 13: RCB Risk Category 4, Stress Scenario. Source: Own diagram.

B.2. Histograms of Simulated CBC Returns

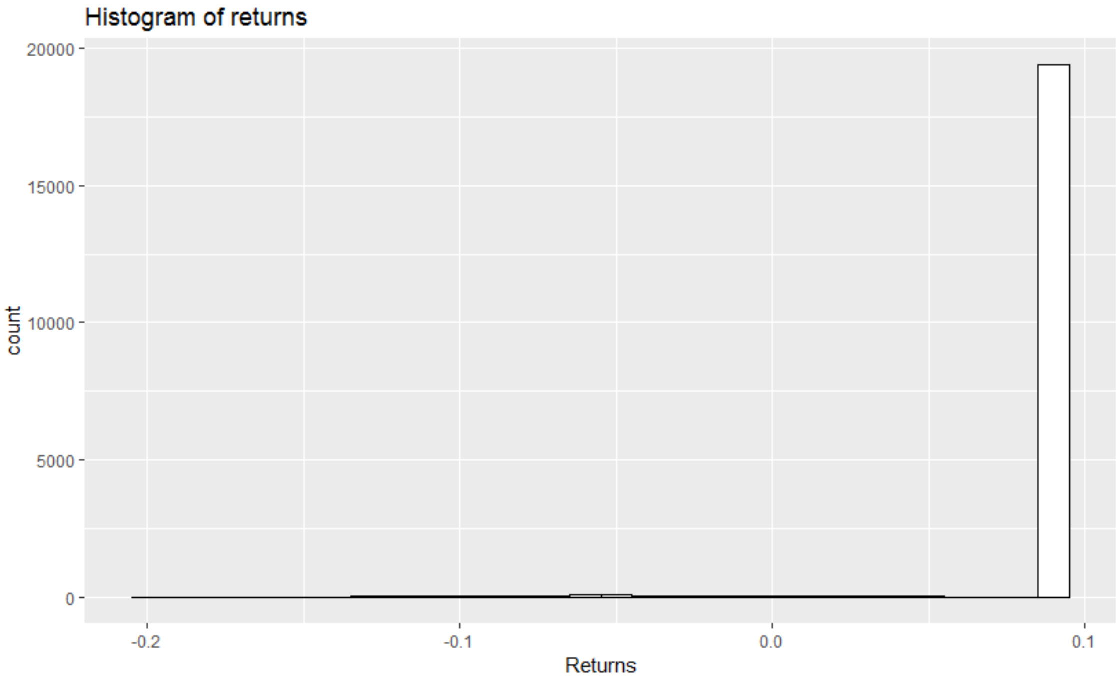

Figure 14: CBC Risk Category 4, Optimistic Scenario. Source: Own diagram.

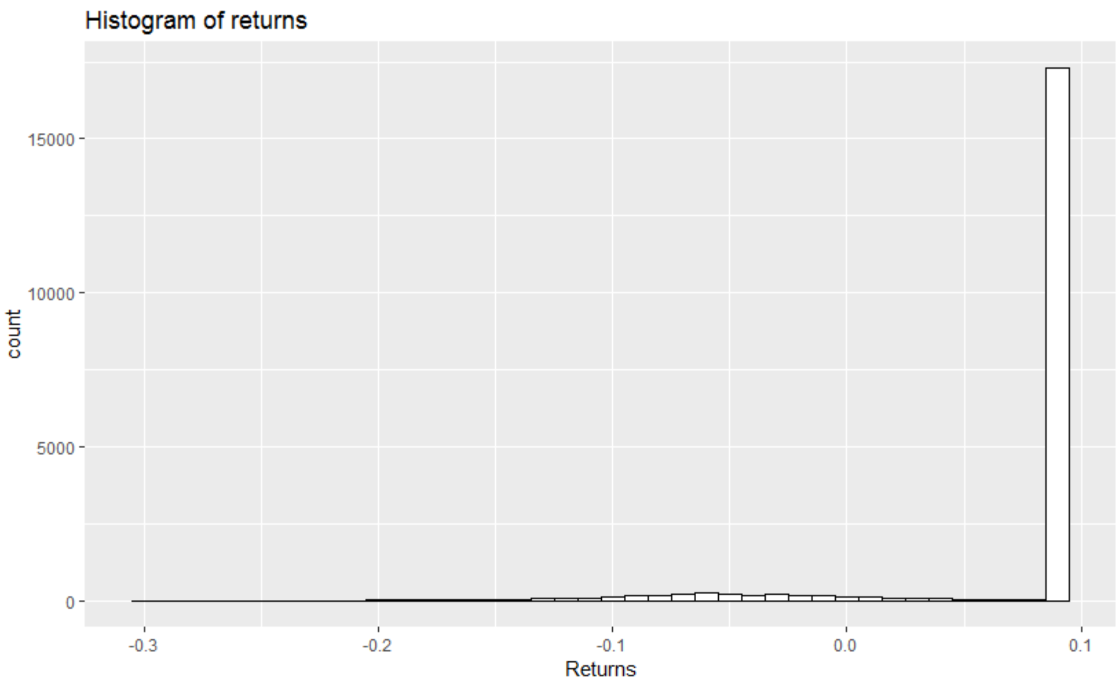

Figure 15: CBC Risk Category 4, Moderate Scenario. Source: Own diagram.

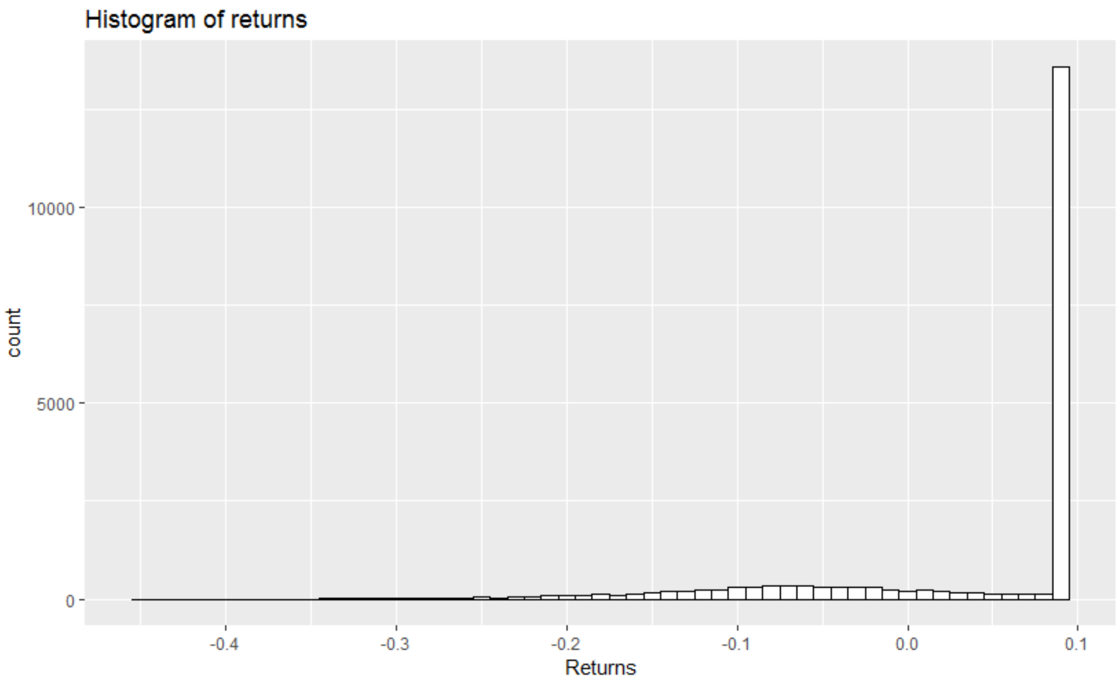

Figure 16: CBC Risk Category 4, Pessimistic Scenario. Source: Own diagram.

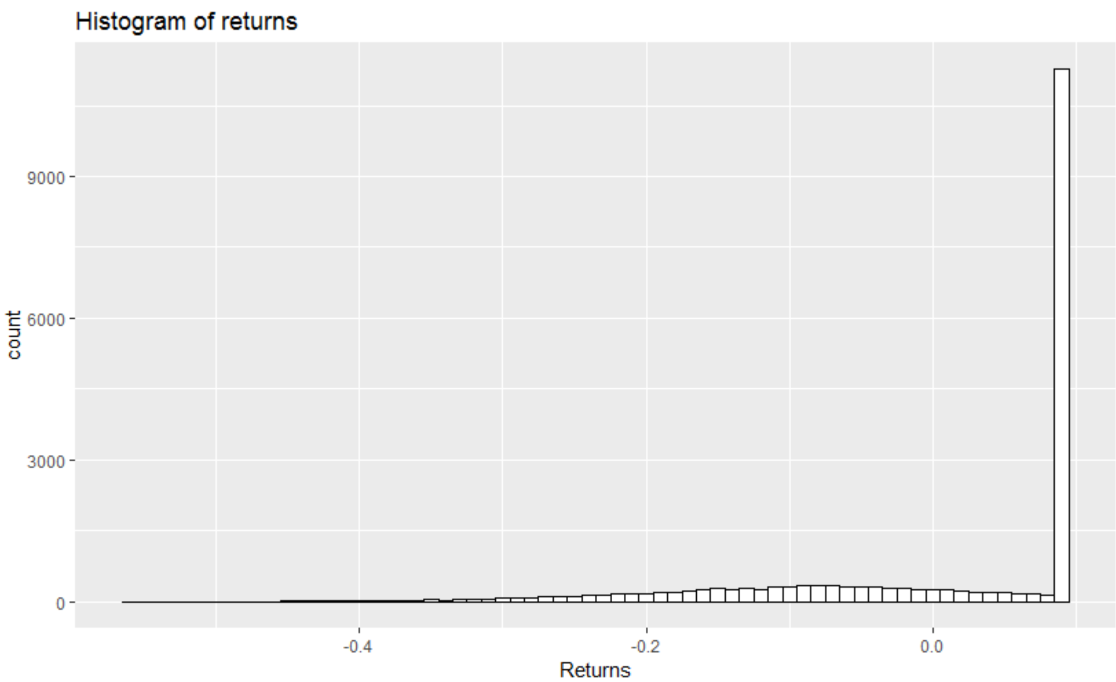

Figure 17: CBC Risk Category 4, Stress Scenario. Source: Own diagram.

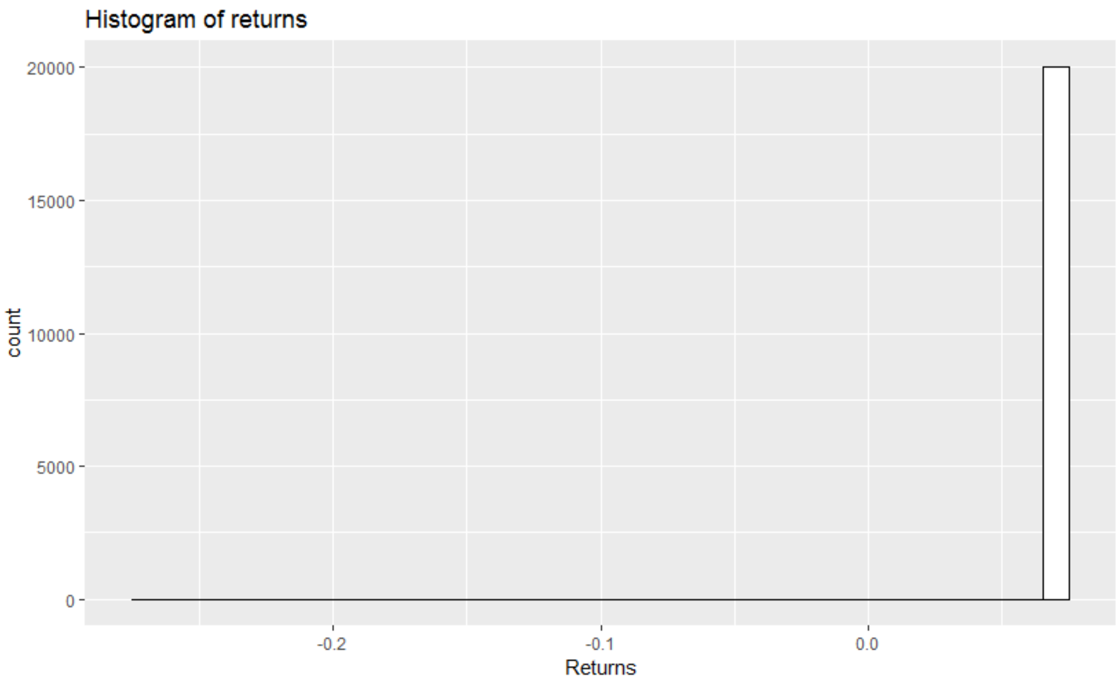

Figure 18: CBC Risk Category 5, Optimistic Scenario. Source: Own diagram.

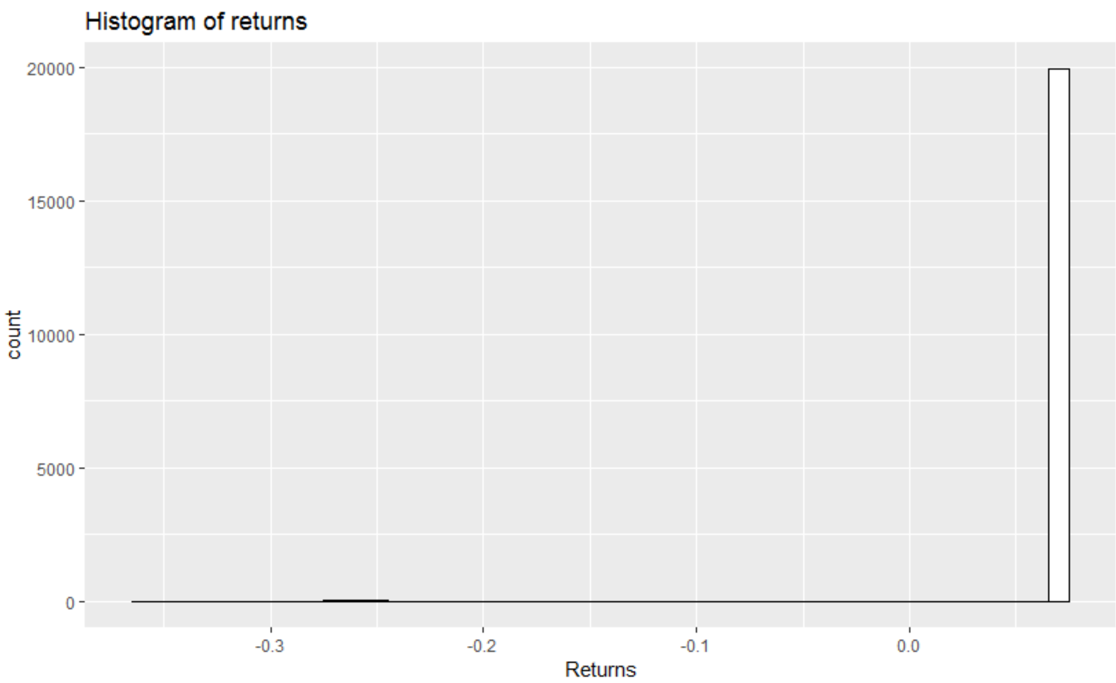

Figure 19: CBC Risk Category 5, Moderate Scenario. Source: Own diagram.

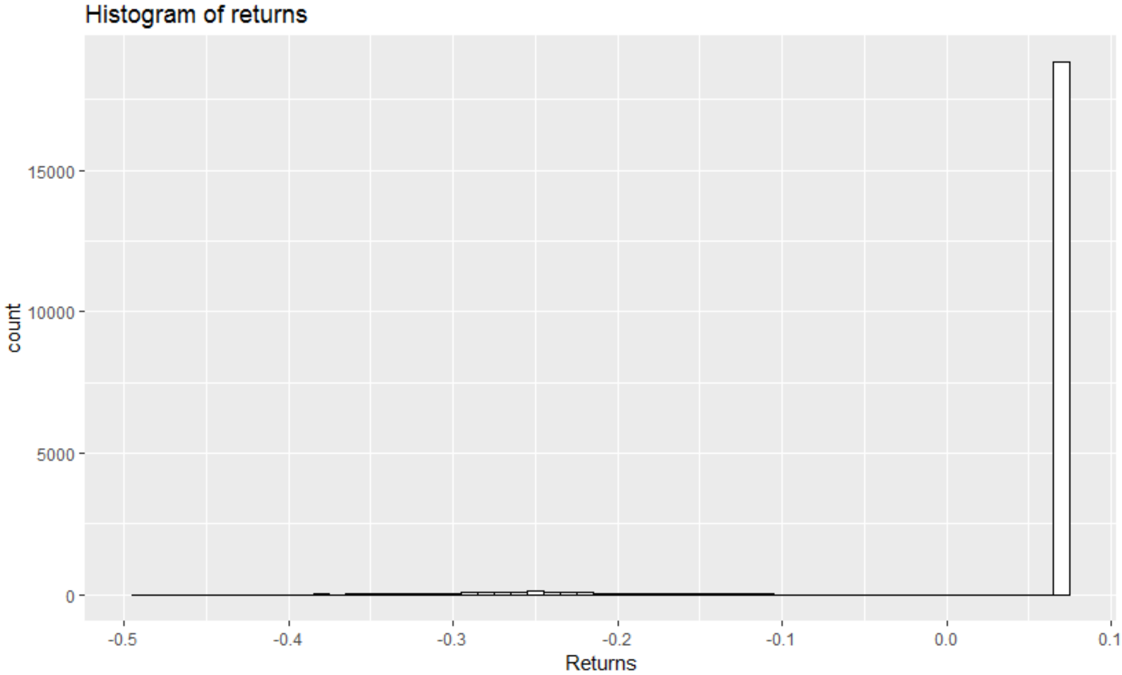

Figure 20: CBC Risk Category 5, Pessimistic Scenario. Source: Own diagram.

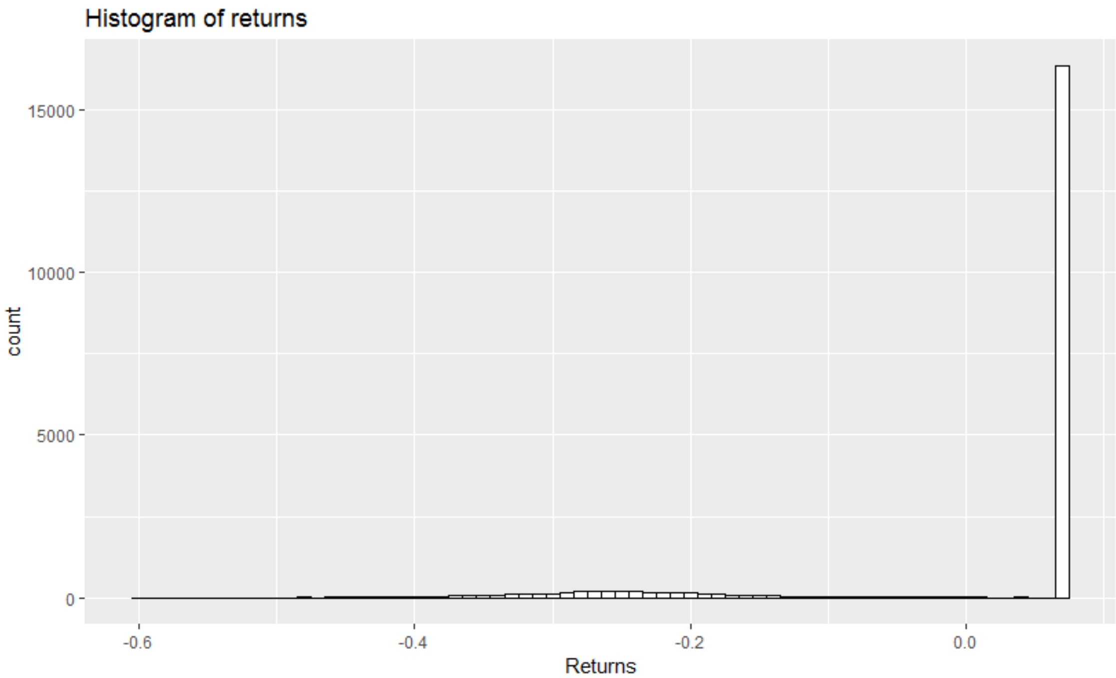

Figure 21: CBC Risk Category 5, Stress Scenario. Source: Own diagram.

Declaration of Authorship

I hereby declare that I have composed my thesis titled ‘Performance Analysis of Structured Financial Instruments for Different Risk Profiles under Various Market Scenarios’ independently using only those resources mentioned and that I have as such identified all passages which I have taken from publications verbatim or in substance. I am informed that my thesis might be controlled by anti-plagiarism software. Neither this thesis nor any extract of it, has been previously submitted to an examining authority, in this or a similar form. I have ensured that the written version of this thesis is identical to the version saved on the enclosed storage medium.

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